P1/P2 HT 2019

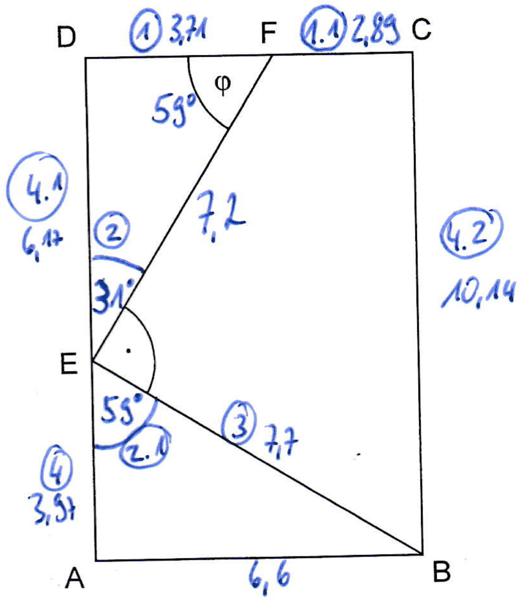
Aufgabe P 1:

(4 P)

Im Rechteck ABCD gilt:

$\overline{AB} = 6{,}6$ cm
$\overline{EF} = 7{,}2$ cm
$\varphi = 59{,}0°$

Berechnen Sie den Umfang des Vierecks EBCF.



① $\overline{DF}$: $\cos(59) = \frac{\overline{DF}}{7{,}2}$
$\overline{DF} = \underline{3{,}71}$

①.① $\overline{CF}$: $\overline{AB} - \overline{DF} = 6{,}6 - 3{,}71 = \underline{2{,}89}$

② ε_1: $\sin(\varepsilon) = \frac{3{,}71}{7{,}2}$
$\varepsilon_1 = \underline{31°}$

②.① ε_2: $180° - 90° - 31° = \underline{59°}$

③ $\overline{BE}$: $\sin(59) = \frac{6{,}6}{\overline{BE}}$
$\overline{BE} = \underline{7{,}70}$

④ $\overline{AE}$: $\cos(59) = \frac{\overline{AE}}{7{,}7}$
$\overline{AE} = \underline{3{,}97}$

④.① $\overline{DE}$: $\cos(31) = \frac{\overline{DE}}{7{,}2}$
$\overline{DE} = \underline{6{,}17}$
$\Rightarrow \overline{BC} = 3{,}97 + 6{,}17$
$= 10{,}14$

$u = \overline{CF} + \overline{EF} + \overline{BE} + \overline{BC} = 2{,}89 + 7{,}2 + 7{,}7 + 10{,}14 = \underline{27{,}93}$ $\underline{\underline{\approx 28 \text{ cm}}}$

Aufgabe P 2:

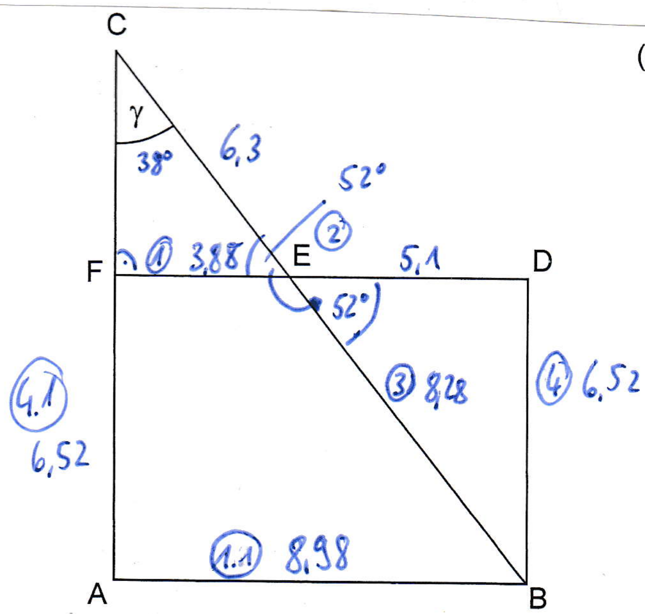
(4 P)

Das Dreieck ABC und das Rechteck ABDF überdecken sich teilweise.

Es gilt:

$\overline{CE} = 6{,}3$ cm
$\overline{DE} = 5{,}1$ cm
$\gamma = 38{,}0°$

Berechnen Sie den Flächeninhalt des Trapezes ABEF.



① $\overline{EF}$: $\sin(38) = \frac{\overline{EF}}{6{,}3}$
$\overline{EF} = 3{,}88$

①.① AB: $\overline{DE} + \overline{EF} = 8{,}98$

② ε_1: $180 - 90 - 38 = 52°$
$\Rightarrow \varepsilon_2 = 52°$

③ $\overline{BE}$: $\cos(52) = \frac{5{,}1}{\overline{BE}}$
$\overline{BE} = 8{,}28$

④ $\sin(52) = \frac{\overline{BD}}{8{,}28}$
$\overline{BD} = 6{,}52$
$\Rightarrow \overline{AF} = 6{,}52$

⑤ $A_{\text{Trapez}} = \frac{(a+c)}{2} \cdot h = \frac{(3{,}88 + 8{,}98)}{2} \cdot 6{,}52 = \underline{\underline{41{,}92 \text{ cm}^2}}$

HT 2019

(3) I. $\frac{x+2}{4} - y = 6 \quad | \cdot 4$

$x + 2 - 4y = 24 \quad | -2$

$x - 4y = 22$

II. $7 - (x - 2y) = y$

$7 - x + 2y = y \quad | -y \; | -7$

$-x + y = -7$

$$\begin{array}{r} x - 4y = 22 \\ -x + y = -7 \\ \hline \hline -3y = 15 \end{array}$$

$y = -5 \quad \Rightarrow \quad -x - (-5) = 7$

$x = 2 \qquad \mathbb{L} = \{(2|-5)\}$

(4)

$W \rightarrow p_2$ denn die Punkte von W liegen auf p_2: $P(1|0)$ 1

$p_2 \rightarrow A$ " der Scheitelpunkt von p_2 liegt bei $(+3|-4)$ 1

$\Rightarrow y = x^2 - 6x + 5 \Rightarrow y = (x-3)^2 - 9 + 5 \Rightarrow y = (x-3)^2 + 4$

Es fehlt B $y = x^2 - 2x + 5$ 1 $\Rightarrow y = (x-1)^2 + 4 \Rightarrow S(1|4)$ [oder Wertetabelle]

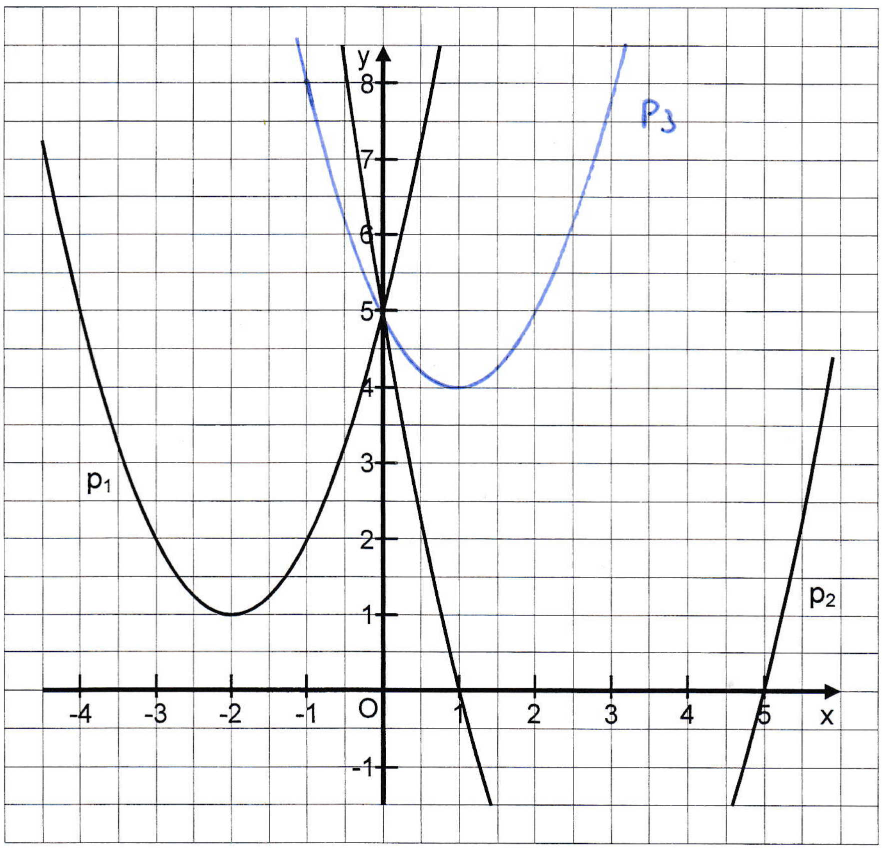
$\Rightarrow$ zeichnen 1

x	0	1	2	3
y	5	0	-3	-4

(A) $y = x^2 - 6x + 5$

(B) $y = x^2 - 2x + 5$

(C) $y = x^2 + 4x + 5$



P5

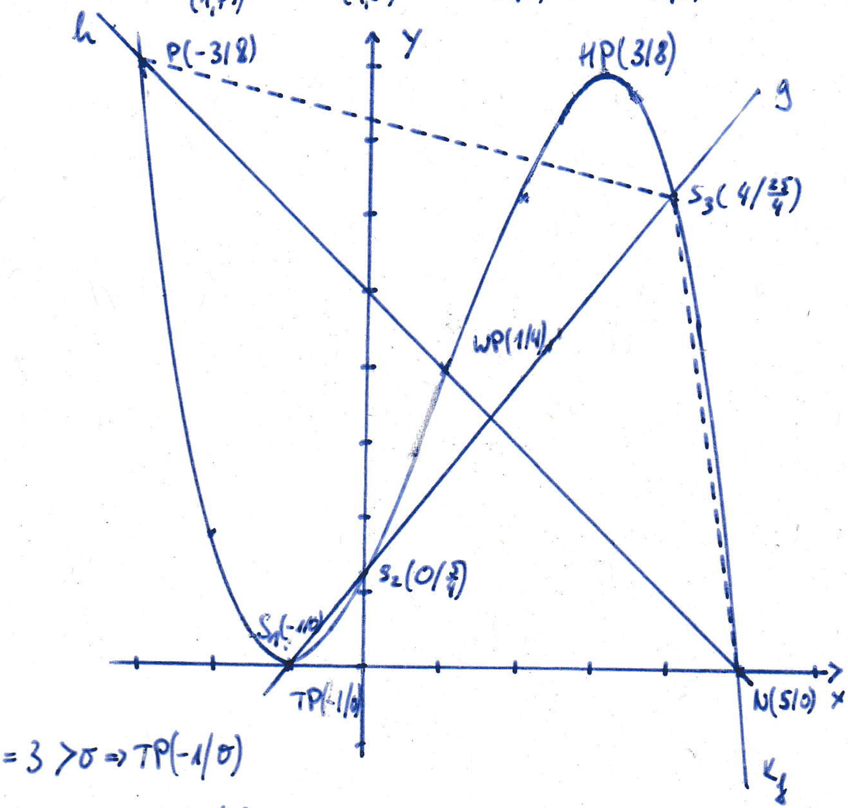
$f(x) = -\frac{1}{4}x^3 + \frac{3}{4}x^2 + \frac{9}{4}x + \frac{5}{4}$

$f'(x) = -\frac{3}{4}x^2 + \frac{3}{2}x + \frac{9}{4}$

$f''(x) = -\frac{3}{2}x + \frac{3}{2}$

$f'''(x) = -\frac{3}{2}$

x	-3	-2	-1	0	1	2	3	4	5
$f(x)$	8	$\frac{7}{4}$ (1,75)	0	$\frac{5}{4}$ (1,25)	4	$\frac{27}{4}$ (6,75)	8	$\frac{25}{4}$ (6,25)	0



EP: n.B.: $f'(x) = 0$

$0 = -\frac{3}{4}x^2 + \frac{3}{2}x + \frac{9}{4} \quad | \cdot (-\frac{4}{3})$

$0 = x^2 - 2x - 3$

$x_{1/2} = 1 \pm \sqrt{1+3}$

$x_1 = 1 - 2 = -1$

$x_2 = 1 + 2 = 3$

h.B.: $f''(x) \neq 0$ $\quad f''(-1) = -\frac{3}{2}(-1) + \frac{3}{2} = 3 > 0 \Rightarrow TP(-1/0)$

$f''(3) = -\frac{3}{2} \cdot 3 + \frac{3}{2} = -3 < 0 \Rightarrow HP(3/8)$

W3 a) g durch $T(-1/0)$; $m = \frac{5}{4}$ $\Rightarrow y = \frac{5}{4} \cdot (x - (-1)) \Rightarrow y = \frac{5}{4}x + \frac{5}{4}$

$K_f \cap g$: $\quad -\frac{1}{4}x^3 + \frac{3}{4}x^2 + \frac{9}{4}x + \frac{5}{4} = \frac{5}{4}x + \frac{5}{4} \quad | \cdot (-4) \quad | -\frac{5}{4}$

$x^3 - 3x^2 - 9x = 5x \quad | -5x$

$x^3 - 3x^2 - 4x = 0$

$x \cdot (x^2 - 3x - 4) = 0 \quad \Rightarrow x = 0 \quad x_{2/3} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}} \Rightarrow x = -1 \quad x = 4$

$S_1(-1/0) \quad S_2(0/\frac{5}{4}) \quad S_3(4/\frac{25}{4})$

WP: $f''(x) = 0 \quad 0 = -\frac{3}{2}x + \frac{3}{2}$

$x = 1$

$f'''(1) = -\frac{3}{2} \neq 0 \Rightarrow WP(1/4)$

h: $N(5/0)$ $WP(1/4)$ $\quad m = \frac{4-0}{1-5} = -1 \quad y = -1 \cdot (x-1) + 4$

$y = -x + 5$

$h \cap K_f$ in $P(-3/8)$: $\quad 8 = -(-3) + 5$

$8 = 8 \Rightarrow P \in h$

Da $P \in K_f$ (siehe Wertetabelle) und $P \in h$, schneidet sich h und K_f in P.

PNS_3 gleichschenklig (?):

$\overline{PS_3} = \sqrt{(6{,}25-8)^2 + (4-(-3))^2} = \sqrt{(\frac{7}{4})^2 + 7^2} = \sqrt{\frac{833}{16}} \approx 7{,}21\,LE$

$\overline{S_3N} = \sqrt{(0-6{,}25)^2 + (5-4)^2} = \sqrt{(-6{,}25)^2 + 1^2} = \sqrt{\frac{641}{16}} \approx 6{,}33\,LE$

Das Dreieck PNS_3 ist nicht gleichschenklig

W3b)

HT 2019 W3b)

$$f(x) = ax^3 + \frac{3}{4}x^2 + \frac{9}{4}x + \frac{5}{4}$$
$$f_a'(x) = 3ax^2 + \frac{3}{2}x + \frac{9}{4}$$
$$f_a''(x) = 6ax + \frac{3}{2}$$
$$f_a'''(x) = 6a$$

WP $\Rightarrow f_a''(x) = 0 \qquad 0 = 6ax + \frac{3}{2} \quad | -\frac{3}{2}$

$$-\frac{3}{2} = 6ax \qquad | : 6a \Rightarrow \cdot \frac{1}{6a}$$
$$-\frac{3}{2} : \frac{6a}{1} = x$$
$$-\frac{3}{2 \cdot 6a} = \underline{\underline{-\frac{1}{4a}}} = x \qquad WP\left(-\frac{1}{4a} \middle| \ \right)$$

Steigung im WP $= 3 \Rightarrow f'(x_{WP}) = 3$

$$3 = 3a \cdot \left(\frac{-1}{4a}\right)^2 + \frac{3}{2} \cdot \left(\frac{-1}{4a}\right) + \frac{9}{4}$$
$$3 = 3a \cdot \frac{1}{16a^2} - \frac{3}{8a} + \frac{9}{4}$$
$$3 = \frac{3}{16a} - \frac{3}{8a} + \frac{9}{4}$$
$$3 = \frac{3}{16a} - \frac{6}{16a} + \frac{9}{4}$$
$$3 = -\frac{3}{16a} + \frac{9}{4}$$
$$\frac{3}{4} = -\frac{3}{16a} \qquad | \cdot 16a$$
$$\frac{3 \cdot \cancel{16}^{4} a}{\cancel{4}} = -3$$
$$12a = -3 \qquad | : 12$$
$$\underline{\underline{a = -\frac{1}{4}}}$$

HT 2019
P6/W4

P6

g_1: $B(-3|4)$ $P(-1{,}5|3{,}5)$

$m = \frac{3{,}5-4}{-1{,}5-(-3)} = \frac{-0{,}5}{1{,}5} = -\frac{1}{3}$

$y = -\frac{1}{3}(x+3)+4$

$y = -\frac{1}{3}x+3$

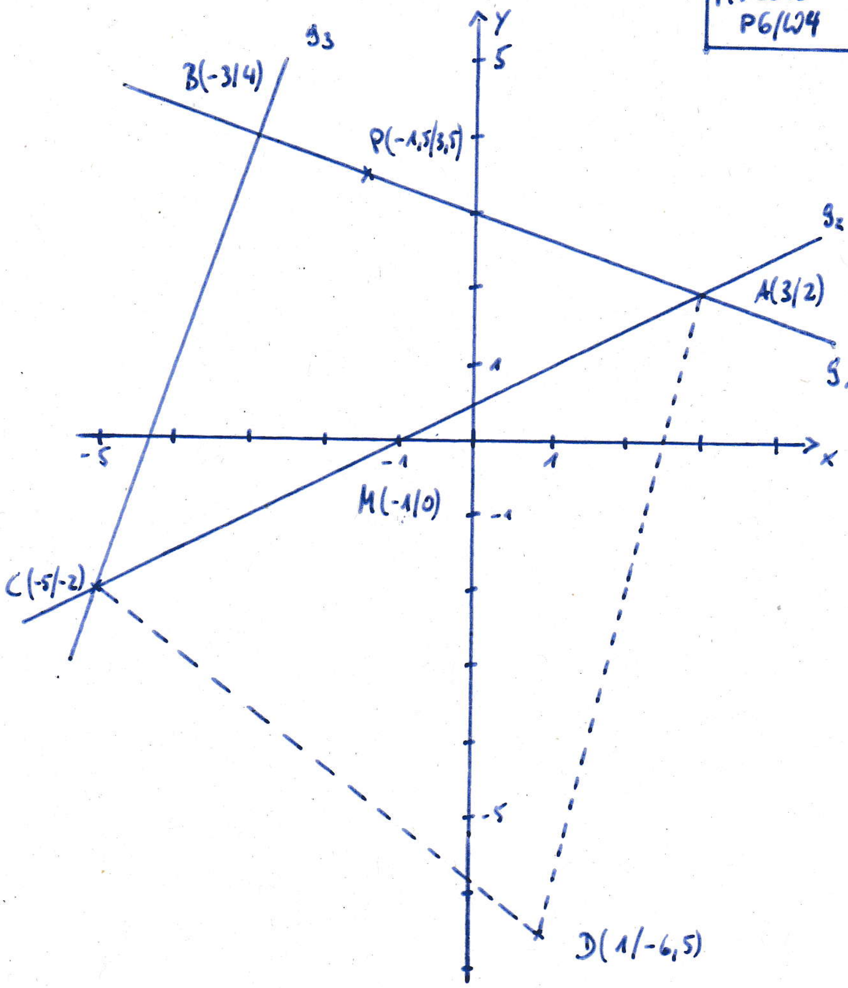
g_2: $2y - x = 1$

$y = \frac{1}{2}x + \frac{1}{2}$

g_3: $m = 3$ $B(-3|4)$

$y = 3(x+3)+4$

$y = 3x + 12$



$C(-5|-2)$ auf g_2: $-2 = \frac{1}{2}\cdot(-5)+\frac{1}{2}$

$-2 = -2 \Rightarrow C \in g_2$

" auf g_3: $-2 = 3\cdot(-5)+12$

$-2 = -2 \Rightarrow C \in g_3$

$g_1 \cap g_2$: $-\frac{1}{3}x+3 = \frac{1}{2}x+\frac{1}{2}$ $\quad |+\frac{1}{3}x \,|-\frac{1}{2}$

$\frac{5}{2} = \frac{5}{6}x$ $\quad |\cdot\frac{6}{5}$

$3 = x$ $\quad \Rightarrow y = \frac{1}{2}\cdot 3+\frac{1}{2} = 2$

$\Rightarrow A(3|2)$

Innenwinkel A: $m_1 = -\frac{1}{3}$ $m_2 = \frac{1}{2}$ $\tan(\alpha) = \frac{-\frac{1}{3}-\frac{1}{2}}{1+\frac{1}{6}} = 1$

$\alpha = 45°$

%: $\frac{60-45}{60}\cdot 100 = 25\%$ Er ist 25% kleiner.

W4

a) Umfang ABC:

$\overline{AB} = \sqrt{(2-4)^2+(3-(-3))^2} = \sqrt{4+36} = \sqrt{40} \approx 6{,}32\,LE$

$\overline{BC} = \sqrt{(4-(-2))^2+(-3-(-5))^2} = \sqrt{36+4} = \sqrt{40} \approx 6{,}32\,LE$

$\overline{AC} = \sqrt{(2-(-2))^2+(3-(-5))^2} = \sqrt{16+64} = \sqrt{80} \approx 8{,}94\,LE$

$u = \sqrt{40}+\sqrt{40}+\sqrt{80} = 21{,}59\,LE$

Mittelpunkt: Da der Satz v. Pythagoras gilt, ist das Dreieck rechtwinklig: $\sqrt{40}^2+\sqrt{40}^2 = \sqrt{80}^2$

$80 = 80$ ✓

$\Rightarrow$ Der Mittelpunkt ist in der Mitte der Hypothenuse:

$M_{AC}\left(\frac{3+(-5)}{2} \middle| \frac{2+(-2)}{2}\right) \Rightarrow M(-1|0)$

Radius: $\overline{M_{AC}A} = \sqrt{(2-0)^2+(3-(-1))^2} = \sqrt{4+16} = \sqrt{20} \approx 4{,}47\,LE = r$

Fläche ΔADC: $C(-5|-2)$ $D(1|-6{,}5)$ $A(3|2)$

$A_\Delta = \frac{1}{2}\left[-5(-6{,}5-2)+1(2+2)+3(-2+6{,}5)\right]$

$A_\Delta = \frac{1}{2}\left[\;42{,}5 + 4 + 13{,}5\;\right]$

$A_\Delta = \frac{1}{2}\cdot 60 = 30\,FE$

Fläche ΔABC: $A = g\cdot h\cdot\frac{1}{2} = \sqrt{40}\cdot\sqrt{40}\cdot\frac{1}{2} = 20\,FE$

Verhältnis $\Delta ADC : \Delta ABC = 30:20 = 3:2$

HT 2013 W4b)

W4 b)

$M(-1|0)$ auf h_k: $y = 2kx + 2k$

$0 = 2k \cdot (-1) + 2k$

$0 = 0 \quad \Rightarrow M$ auf h_k

B auf h_k: $B(-3|4)$ $\quad 4 = 2k \cdot (-3) + 2k$

$4 = -6k + 2k$

$4 = -4k \quad | : -4$

$k = -1$

Wenn $k = -1$ ist, liegt B auf h_k

$h_k \cap g_3$: $\quad 2kx + 2k = 3x + 13 \quad | -3x \; | -2k$

$2kx - 3x = 13 - 2k$

$x(2k - 3) = 13 - 2k \quad | : (2k-3)$

$x = \frac{13-2k}{2k-3}$

S auf der y-Achse: $\Rightarrow x = 0$

$0 = \frac{13-2k}{2k-3} \quad | \cdot (2k-3)$

$0 = 13 - 2k \quad | +2k$

$2k = 13 \quad | : 2$

$k = 6{,}5$

W2 a) HT 2019

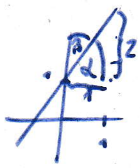
p_1: $S_1(2/2) \Rightarrow y=(x-2)^2+2$
$y = x^2-4x+6$

p_2: $N_1(-2/0)$ $N_2(2/0)$
$y=-(x-2)(x+2) = -(x^2-4)$
$y=-x^2+4$

T: $p_1 \cap p_2$ $\quad x^2-4x+6 = -x^2+4 \quad |+x^2|-4$
$2x^2-4x+2=0 \quad |:2$
$x^2-2x+1=0 \Rightarrow x_{1/2} = 1 \pm \sqrt{0}$ $\quad T(1/3)$ $\quad \rightarrow$ da $y=-1^2+4=3$

g: $m=2$ $T(1/3) \Rightarrow y=2(x-1)+3 \Rightarrow y=2x+1$

Winkel mit y-Achse:



α: $\tan(2) = 63{,}4°$
β: $90-63{,}4 = 26{,}6°$

Parabel p_3: z.B. $S_3(2/5) \Rightarrow y=(x-2)^2+5 \Rightarrow y=x^2-4x+9$

b) p_1: $S_1(0/6)$ $\quad y=ax^2+c \Rightarrow 6=a0^2+c \Rightarrow c=6$
$B(2/4) \Rightarrow 4=a2^2+6$
$-2=4a \Rightarrow a=-0{,}5$
$\Rightarrow y=-\frac{1}{2}x^2+6$

p_2: $B(2/4)$ $\quad y=x^2+3x+q \Rightarrow 4=2^2+3\cdot2+q \Rightarrow q=-6$ $\quad y=x^2+3x-6$

A: $p_1 \cap p_2$: $\quad -\frac{1}{2}x^2+6 = x^2+3x-6 \quad |+\frac{1}{2}x^2|-6$
$0 = \frac{3}{2}x^2+3x-12 \quad |\cdot\frac{2}{3}$
$0 = x^2+2x-8 \Rightarrow x_{1/2} = -1 \pm \sqrt{1+8} \Rightarrow x_1=-4 \quad x_2=2$
$\Downarrow$ $A(-4/-2)$ $\quad \Downarrow$ $B(2/4)$

Gerade: $A(-4/-2)$ $B(2/4)$
$m = \frac{4+2}{2+4} = 1 \Rightarrow y = 1(x-2)+4 \Rightarrow y=x+2$
$\Rightarrow C(0/2)$ liegt auf g: $2=0+2$ ✓