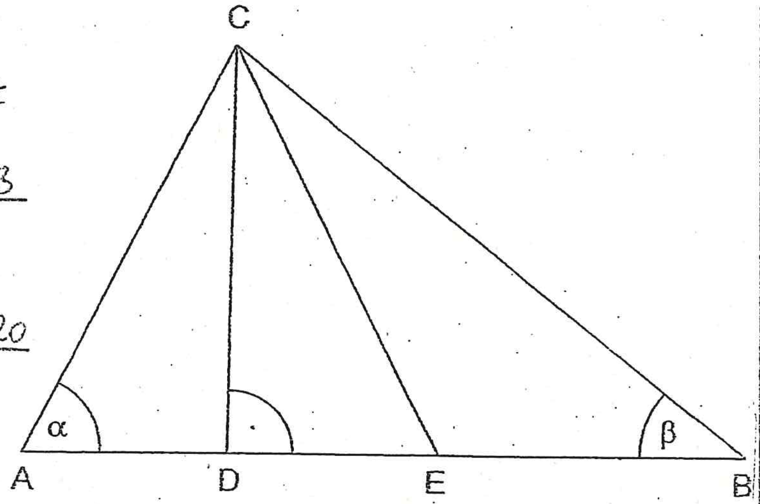
RSA
HT 2015

(P1) gesucht: $U_{\Delta EBC}$

ΔADC $\quad \overline{CD}$: $\sin(64°) = \frac{\overline{CD}}{9,2}$

$\underline{8,27 = \overline{CD}}$



ΔCDE $\quad \overline{DE}$: $\tan(64°) = \frac{8,27}{\overline{DE}}$

$\underline{\overline{DE} = 4,03}$

$\overline{EC}$: $\sin(64) = \frac{8,27}{\overline{CE}}$

$\underline{\overline{CE} = 9,20}$

ΔBCD $\quad \overline{BC}$: $\sin(40) = \frac{8,27}{\overline{BC}}$

$\underline{\overline{BC} = 12,87}$

$\overline{BD}$: $\tan(40) = \frac{8,27}{\overline{BD}}$ $\quad \Rightarrow \overline{BE} = \overline{BD} - \overline{DE} = \underline{5,83}$

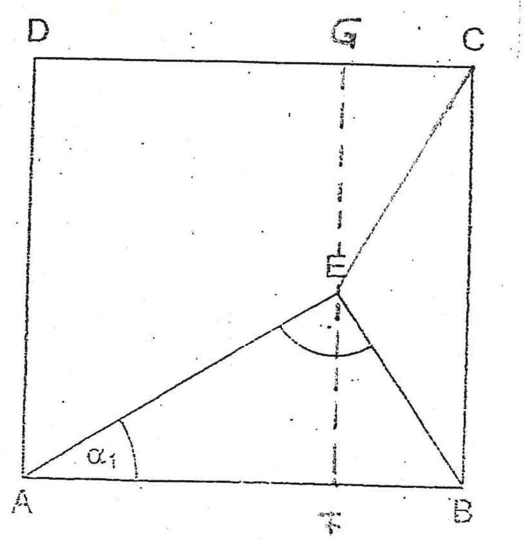
$\underline{\overline{BD} = 9,86}$

$U_{EBC} = \overline{BC} + \overline{CE} + \overline{BE} = 12,87 + 9,2 + 5,83 = \underline{\underline{27,9\text{ cm}}}$

(P2)

ΔABE: $\quad \overline{AB}$: $\cos(34) = \frac{7,8}{\overline{AB}}$

$\underline{\overline{AB} = 9,41}$



ΔAFE $\quad \overline{EF}$: $\sin(34) = \frac{\overline{EF}}{7,8}$

$\underline{4,36 = \overline{EF}}$

$\Rightarrow \overline{EG} = 9,41 - 4,36 = \underline{5,05}$

$\overline{AF}$: $\tan(34) = \frac{4,36}{\overline{AF}}$

$\underline{\overline{AF} = 6,46}$

$\Rightarrow \overline{BF} = 9,41 - 6,46 = \underline{2,95}$

$\Rightarrow \overline{CG} = 2,95$

ΔECG $\quad a^2 + b^2 = c^2$

$5,05^2 + 2,95^2 = c^2$

$\underline{\underline{5,85 = c}}$

(P1)

RSA HT 2015 P_1 P_2

$\triangle ADC$

$\sin\alpha = \frac{\overline{CD}}{\overline{AC}} \Rightarrow$

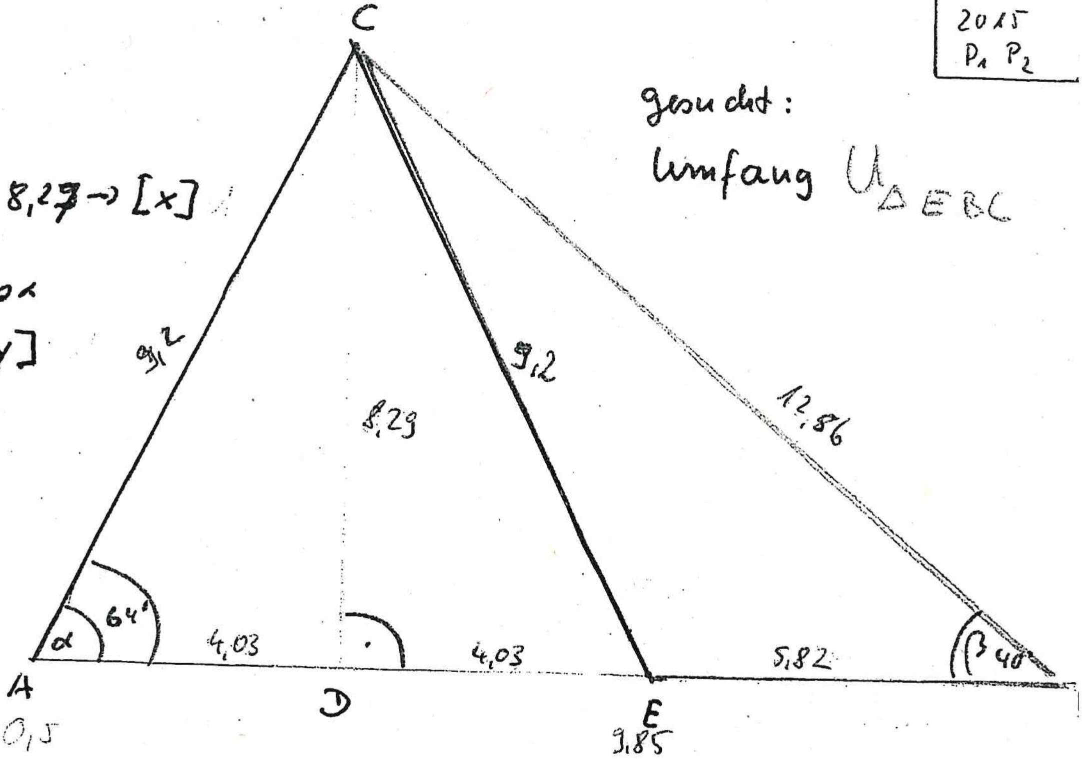
$\overline{CD} = 9{,}2 \cdot \sin 64° = 8{,}27 \rightarrow [x]$

Strecke $\overline{AD}$: Pyth/cos α

$\overline{AD} = 4{,}03 \rightarrow [y]$

gesucht:

Umfang $U_{\triangle EBC}$



$\triangle BCD$

Strecke $\overline{BC}$:

$\sin\beta = \frac{\overline{CD}}{\overline{BC}} \Rightarrow$

$\overline{BC} = \frac{8{,}29}{\sin 40}$

$\overline{BC} = 12{,}86 \rightarrow [z]$ 0,5

Strecke $\overline{BD}$: Pyth/cos β, tan β $\Rightarrow \overline{BD} = 9{,}85 \rightarrow [t]$ 0,5

$\overline{EB} = \overline{BD} - \overline{AD} = 5{,}82 \rightarrow [a]$ 0,5

$\Rightarrow U = \overline{EB} + \overline{BC} + \overline{CE} = 5{,}82 + 12{,}86 + 9{,}2 = 27{,}89$

$U = 27{,}9\,cm$ 0,5

(P2)

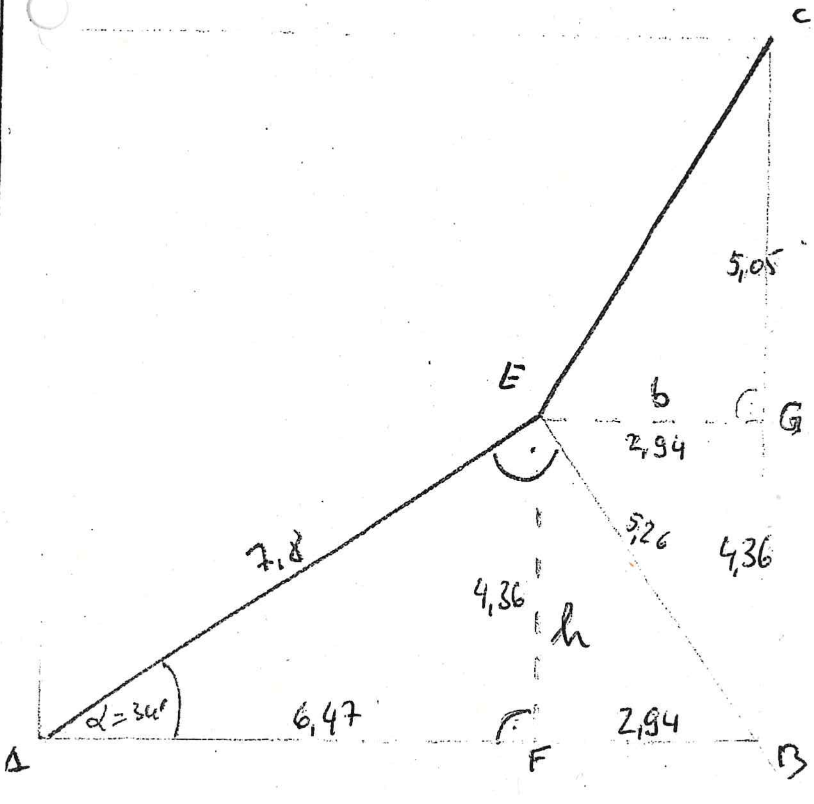
$\triangle ABE$ Strecke $\overline{AB}$: $\cos\alpha = \frac{\overline{AE}}{\overline{AB}} \Rightarrow \overline{AB} = \frac{7{,}8}{\cos 34°} = 9{,}41 \rightarrow [x]$ 1

Strecke $h = \overline{EF}$: $\sin 34° = \frac{h}{7{,}8} \Rightarrow h = 7{,}8 \cdot \sin 34° = 4{,}36 \rightarrow [y]$ 0,5

Strecke $\overline{AF}$: $\overline{AF} = \sqrt{7{,}8^2 - h^2}$

$\overline{AF} = 6{,}47 \rightarrow [z]$ 1

$\Rightarrow b = \overline{FB} = \overline{AB} - \overline{AF} = 2{,}94 \rightarrow [t]$ 0,5



$\triangle EGC$

Strecke $\overline{CG}$: $\overline{CG} = \overline{AB} - h$

$\overline{CG} = 9{,}41 - 4{,}36 = 5{,}05 \rightarrow [a]$ 0,5

$\overline{EC}$ Pyth: $\overline{EC} = \sqrt{\overline{CG}^2 + b^2} = 5{,}84$

$\overline{EC} = 5{,}84\,cm$

RSA
2015
NT
P3, P4
W2

P3 I $\frac{2x}{3} + y = 5 \mid \cdot 3 \Rightarrow 2x + 3y = 15$

II $6x + 3y - 6y - 2x = 21$
$\Rightarrow 4x - 3y = 21$

$$\left.\begin{aligned} 2x + 3y &= 15 \\ 4x - 3y &= 21 \\ \hline 6x &= 36 \end{aligned}\right.$$

$\Rightarrow \underline{x = 6}$

$\mathbb{L} = \{(6 \mid 1)\}$ einsetzen $\Rightarrow \underline{y = 1}$

P4 p_1 über Nullstellen $\Rightarrow y = (x+7)(x+3) \Rightarrow \underline{p_1: y = x^2 + 10x + 21}$

p_2 über Scheitelpunkt $\Rightarrow y = (x-3)^2 + 4 \Rightarrow \underline{p_2: y = x^2 - 6x + 13}$

Punktprobe: $A \in p_1$: $21 = 0 + 0 + 21$
$B \in p_1$: $12 = 1 - 10 + 21$ also $\underline{A, B \in p_1}$

Schnittpunkt Q: $x^2 + 10x + 21 = x^2 - 6x + 13$
$16x = -8 \Rightarrow x = -\frac{1}{2} \Rightarrow y = 16{,}25 \Rightarrow \underline{Q\left(-\frac{1}{2} \mid \frac{65}{4}\right)}$

W2 a) $Q(3 \mid -3) \in p_1 \Rightarrow -3 = 9a + 3 \Rightarrow a = -\frac{2}{3} \Rightarrow \underline{p_1: y = -\frac{2}{3}x^2 + 3}$

$\in g \Rightarrow -3 = -6 + b \Rightarrow b = 3 \Rightarrow \underline{g: y = -2x + 3}$

$x = 0 \Rightarrow y = 3 \Rightarrow \underline{P(0 \mid 3)}$ ist der 2. Schnittpunkt.

$Q \in p_2: y = x^2 + px \Rightarrow -3 = 9 + 3p \Rightarrow p = -4 \Rightarrow \underline{p_2: y = x^2 - 4x}$

Scheitelpunkt: $y = (x-2)^2 - 4 \Rightarrow \underline{S_2(2 \mid -4)}$

Strecke $\overline{PS_2}$: $d = \sqrt{(2-0)^2 + (-4-3)^2} = \underline{\sqrt{53} = \overline{PS_2} \approx 7{,}3 \text{ LE}}$

b) $p_1: y = x^2 - 6x + 5$
$= (x-5)(x-1)$
$= (x-3)^2 - 4$

$\Rightarrow \underline{N_1(1 \mid 0); N_2(5 \mid 0) \quad S_y = A(0 \mid 5)}$

$S_1(3 \mid -4) \Rightarrow S_2(3-4 \mid -4+5)$

$\Rightarrow \underline{p_2: y = (x+1)^2 + 1 = x^2 + 2x + 2}$

$g: y = 5$

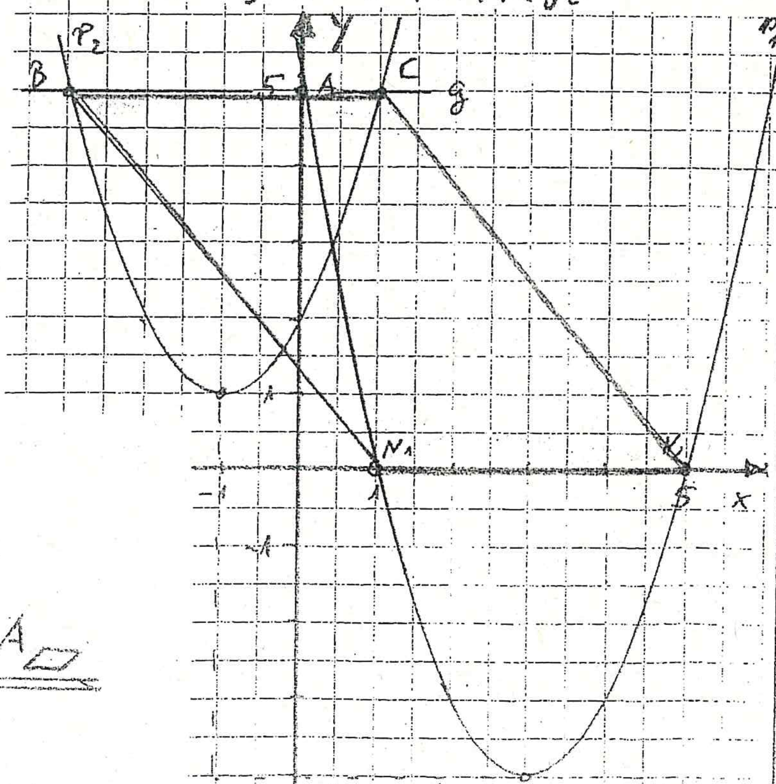
$\Rightarrow x^2 + 2x + 2 = 5 \Rightarrow x^2 + 2x - 3 = 0$

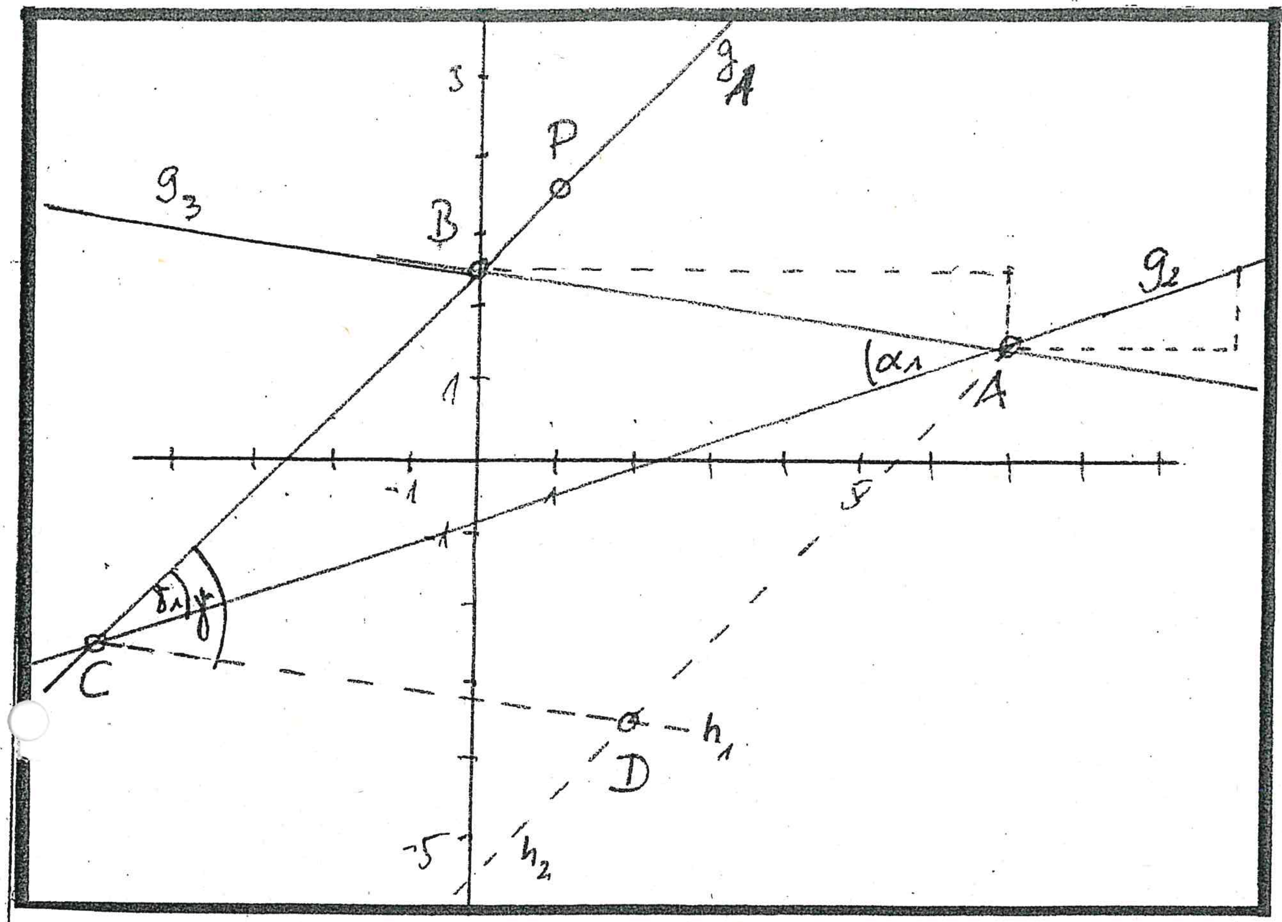
$x_{1/2} = -1 \pm \sqrt{1+3} = -1 \pm 2$

$\Rightarrow \underline{B(-3 \mid 5); C(1 \mid 5)}$

$A = \overline{N_1N_2} \cdot y_B = 4 \cdot 5 = \underline{20 \text{ FE} = A_{▱}}$

Zeichnung nicht verlangt





RSA
HT
2015
P6
W4a/b

Ergebnisse

$g_1: y=x+\frac{5}{2}$

$g_2: y=\frac{1}{3}x-\frac{5}{6}$

$g_3: y=-\frac{1}{7}x+\frac{5}{2}$

$C(-5|-\frac{5}{2})$

Gerade g_1: $y=\frac{3{,}5-2{,}5}{1-0}\cdot(x-0)+2{,}5 = 1x+2{,}5 = x+\frac{5}{2}$ $\quad \underline{\underline{g_1: y=x+\frac{5}{2}}}$

Gerade g_2: $m=+\frac{1}{3} \Rightarrow y=\frac{1}{3}(x-7)+\frac{3}{2} = \frac{1}{3}x-\frac{5}{6}$ $\quad \underline{\underline{g_2: y=\frac{1}{3}x-\frac{5}{6}}}$

Schnittpunkt C: $x+\frac{5}{2}=\frac{1}{3}x-\frac{5}{6} \Rightarrow \frac{2}{3}x=-\frac{10}{3} \Rightarrow x=-5 \Rightarrow y=-\frac{5}{2} \Rightarrow \underline{\underline{C(-5|-\frac{5}{2})}}$

A auf g_3: Punktprobe: $\frac{3}{2}=-\frac{1}{7}\cdot 7+\frac{5}{2}=\frac{3}{2} \Rightarrow \underline{\underline{A\in g_3}}$

Längen: $\overline{CP}=\sqrt{(1+5)^2+(3{,}5+2{,}5)^2}=\sqrt{72}$

$\overline{AP}=\sqrt{(7-1)^2+(1{,}5-3{,}5)^2}=\sqrt{40}$

Prozent: $\frac{\overline{CP}-\overline{AP}}{\overline{AP}}\cdot 100\,\% = \frac{\sqrt{72}-\sqrt{40}}{\sqrt{40}}\cdot 100 = \underline{\underline{34{,}2\,\%}}$

Die Strecke $\overline{CP}$ ist um 34,2% länger als die Strecke $\overline{AP}$.

(W 4a) Gleichschenkligkeit

Version: Winkel

$$\tan\gamma_1 = \frac{1-\frac{1}{3}}{1+1\cdot\frac{1}{3}} = \frac{1}{2} \Rightarrow \gamma_1 = 26{,}565^\circ$$

gleiche Basiswinkel

$$\tan\alpha_1 = \frac{\frac{1}{3}+\frac{1}{7}}{1+\frac{1}{3}\left(-\frac{1}{7}\right)} = \frac{1}{2} \Rightarrow \alpha_1 = 26{,}565^\circ$$

Version: Längen

$$\overline{CB} = \sqrt{(0+5)^2+(2{,}5+2{,}5)^2} = \sqrt{50}$$

$$\overline{AB} = \sqrt{(7-0)^2+(1{,}5-2{,}5)^2} = \sqrt{50}$$

gleich lange Seiten

*)

Punkt D für Raute

klassischer Weg

Version: parallele Geraden:

$$h_1: C,\ m=-\frac{1}{7} \Rightarrow y = -\frac{1}{7}(x+5)-\frac{5}{2} \Rightarrow h_1: y = -\frac{1}{7}x-\frac{45}{14}$$

$$h_2: A;\ m=1 \Rightarrow y = 1(x-7)+\frac{3}{2} \Rightarrow h_2: y = x-\frac{11}{2}$$

Schnitt: $-\frac{1}{7}x-\frac{45}{14} = x-\frac{11}{2} \Rightarrow x=2 \Rightarrow y=-\frac{7}{2} \Rightarrow \underline{\underline{D\left(2\middle|-\frac{7}{2}\right)}}$

Version Verschiebung

Vektor

$B \to A$ Verschiebung um $+7$ in x-Richtung
Verschiebung um -1 in y-Richtung

$$C \to D: \left.\begin{array}{l} x_C+7 = x_D = -5+7 = 2 \\ y_C-1 = y_D = -\frac{5}{2}-1 = -\frac{7}{2} \end{array}\right\} \underline{\underline{D = \left(2\middle|-\frac{7}{2}\right)}}$$

Winkel γ

$$\gamma = 2\cdot\gamma_1 = 53{,}13^\circ$$

oder $\tan\gamma = \dfrac{1+\frac{1}{7}}{1+1\left(-\frac{1}{7}\right)} = \dfrac{4}{3} \Rightarrow \gamma = 53{,}13^\circ$

Fläche Raute

Version: $A = \frac{1}{2}e\cdot f$

$$e = \overline{AC} = \sqrt{(7+5)^2+(1{,}5+2{,}5)^2} = \sqrt{160}$$

$$f = \overline{BD} = \sqrt{(2-0)^2+\left(-\frac{7}{2}-\frac{5}{2}\right)^2} = \sqrt{40}$$

$$\Rightarrow \underline{A} = \frac{1}{2}\cdot\sqrt{160}\cdot\sqrt{40} = \underline{\underline{40\,FE}}$$

Version: $A = 2A_{\triangle ABC}$

$$A_{\triangle ABC} = \frac{1}{2}\cdot\left[7\cdot(2{,}5+2{,}5)+0 * 5\,(1{,}5-2{,}5)\right] = 20\,FE$$

$$\Rightarrow \underline{\underline{A = 40\,FE}}$$

*) Im Text war die Orientierung ABCD vorgegeben. Sonst wären auch die Punkte $D_2\left(12\middle|\frac{13}{2}\right)$; $D_3\left(-10\middle|-\frac{3}{2}\right)$ denkbar mit anderen Werten für γ.

(W4b) $h_m: y = m\cdot x - m - \frac{1}{2}$ $\qquad g: y = x - \frac{1}{2}$ $\qquad M(1|-\frac{1}{2})$

$M \in h_m: \quad -\frac{1}{2} = 1\cdot m - m - \frac{1}{2} \Rightarrow 0 = 0$ $\quad$ <u>also liegt M auf allen h_m</u>

Schnittstelle

$$x - \frac{1}{2} = m\cdot x - m - \frac{1}{2}$$

$$x - mx = -m$$

$$x(1-m) = -m \qquad | : (1-m) \qquad (\text{Anm. } m \neq 1)$$

$$\underline{\underline{x = \frac{-m}{1-m}}} \quad \text{oder} \quad \underline{\underline{x = \frac{m}{m-1}}}$$

$$x = -\frac{1}{2} = \frac{-m}{1-m} \Rightarrow 1 - m = 2m$$

$$1 = 3m \Rightarrow \underline{\underline{m = \frac{1}{3}}}$$

Punktverteilung:

(P6) Zeichnung:

g_1	0,5 P
g_2	1 P
g_3	0,5 P

Rechnung:

g_1	1 P
g_2	1 P
C	1 P
A	0,5 P
$\overline{CP}$	0,5 P
$\overline{AP}$	0,5 P
[illegible]	1 P

(W4a)

gleichschenklig	1 P
D	2,5 P
Winkel γ	1 P
Fläche A	2 P

(W4b)

$M \in h_m$	1
x_m	1,5
$m = \frac{1}{3}$	1

$$f(x) = \frac{1}{8}x^3 - \frac{3}{8}x^2 - \frac{9}{8}x + \frac{27}{8}$$

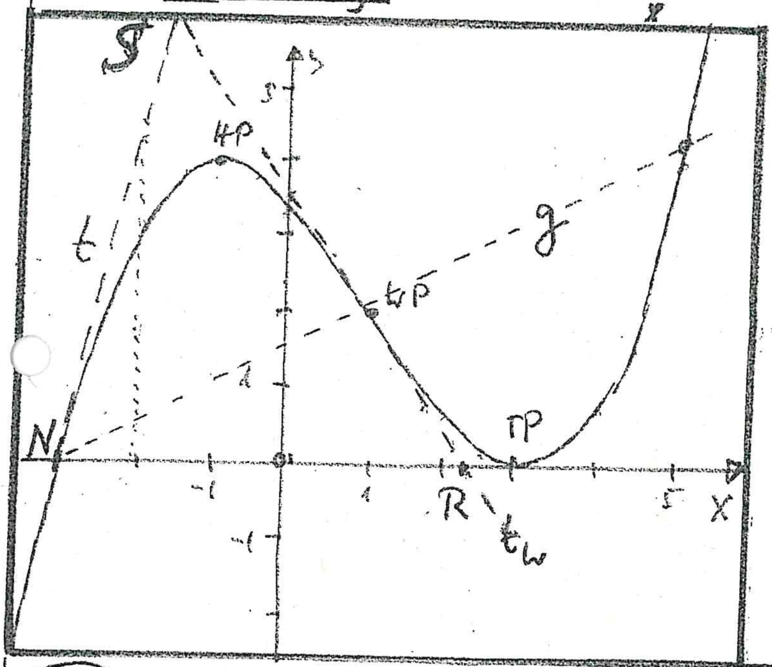
RSA
HT
2015
P5
W3 a/b

P5 Ableitungen: $f'(x) = \frac{3}{8}x^2 - \frac{3}{4}x - \frac{9}{8}$ $\qquad f''(x) = \frac{3}{4}x - \frac{3}{4}$

Wertetabelle:

x	-3	-2	-1	0	1	2	3	4	5
f(x)	0	3,1	4	3,4	2	0,6	0	0,9	4
		$\frac{25}{8}$		$\frac{27}{8}$		$\frac{5}{8}$		$\frac{7}{8}$	

Zeichnung



Extremstellen/-punkte

n.B. $f'(x) = 0 \Rightarrow \frac{3}{8}x^2 - \frac{3}{4}x - \frac{9}{8} = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x_{1/2} = 1 \pm \sqrt{1+3} \Rightarrow x_1 = -1$; $x_2 = 3$

hinr. Bed: $f''(-1) = -\frac{3}{2} < 0 \Rightarrow HP$

$f(-1) = 4 \Rightarrow \underline{\underline{HP(-1|4)}}$

$f''(3) = +\frac{3}{2} > 0 \Rightarrow TP$

$f(3) = 0 \Rightarrow \underline{\underline{TP(3|0)}}$

W3a Tangente t in $N(-3|y)$ $\quad f(-3) = 0 \Rightarrow N(-3|0) \quad f'(-3) = 4,5 = \frac{9}{2}$

$\Rightarrow$ Punkt-Steigungsform: $y = \frac{9}{2}(x+3) \Rightarrow \underline{\underline{t: y = \frac{9}{2}x + \frac{27}{2}}}$

Wendetangente: WP: $f''(x) = 0 = \frac{3}{4}x - \frac{3}{4} \Rightarrow x = 1 \Rightarrow WP(1|2)$

$f'(1) = -\frac{3}{2} \Rightarrow y = -\frac{3}{2}(x-1) + 2 \Rightarrow \underline{\underline{t_w: y = -\frac{3}{2}x + \frac{7}{2}}}$

Schnittpunkt S $\quad \frac{9}{2}x + \frac{27}{2} = -\frac{3}{2}x + \frac{7}{2} \Rightarrow 6x = -10 \Rightarrow x = -\frac{5}{3}$

in t oder $t_w \Rightarrow y = 6 \Rightarrow \underline{\underline{S(-\frac{5}{3}|6)}}$

Schnittpunkt R $\quad t_w: y = 0 \Rightarrow -\frac{3}{2}x + \frac{7}{2} = 0 \Rightarrow x = \frac{7}{3} \Rightarrow \underline{\underline{R(\frac{7}{3}|0)}}$

*) Fläche $A_{\triangle NWS}$: $\underline{\underline{A_1}} = \frac{1}{2}\left[-3(2-6) + 1(6-0) + (-\frac{5}{3})(0-2)\right] = \underline{\underline{\frac{32}{3} FE}}$

Fläche $A_{\triangle NRW}$: $\underline{\underline{A_2}} = \frac{1}{2} \cdot (\frac{7}{3} + 3) \cdot 2 = \underline{\underline{\frac{16}{3} FE}}$

Prozent: $\frac{A_2}{A_1} \cdot 100 = \frac{16/3}{32/3} \cdot 100 = 50 \Rightarrow$ Flächenanteil ist 50 %

Gerade g: 2 Punkteform: $N(-3|0); W(1|2) \Rightarrow y = \frac{2-0}{1+3}(x-1) + 2$

$\Rightarrow \underline{\underline{g: y = \frac{1}{2}x + \frac{3}{2}}}$

Schnittwinkel $\tan\varphi = \frac{\frac{1}{2} - (-\frac{3}{2})}{1 + \frac{1}{2}(-\frac{3}{2})} = 8 \Rightarrow \underline{\underline{\varphi = 82,9°}}$

*) $A_{NRS} = \frac{1}{2} \cdot (\frac{7}{3} - (-3)) \cdot (6-0) = 16 FE$
$A_2 = \frac{1}{2} \cdot (\frac{7}{3} - (-\frac{5}{3})) \cdot 2 = \frac{16}{3}$
$A_1 = \frac{32}{3}$

W3b) Vorgegeben $J = [1;5]$

hier verläuft g oberhalb von K_f

$$\left.\begin{array}{l} P(u \mid f(u)) \in K_f \\ Q(u \mid g(u)) \in g \end{array}\right\} \Rightarrow d(u) = g(u) - f(u)$$

$$= \left(\frac{1}{2}u + \frac{3}{2}\right) - \left(\frac{1}{8}u^3 - \frac{3}{8}u^2 - \frac{9}{8}u + \frac{27}{8}\right)$$

$$\underline{\underline{d(u) = -\frac{1}{8}u^3 + \frac{3}{8}u^2 + \frac{13}{8}u - \frac{15}{8}}}$$

Extremal:

$$d'(u) = -\frac{3}{8}u^2 + \frac{3}{4}u + \frac{13}{8} = 0 \quad \left| \cdot \left(-\frac{8}{3}\right)\right.$$

$$u^2 - 2u - \frac{13}{3} = 0$$

$$\Rightarrow u_{1/2} = +1 \pm \sqrt{1 + \frac{13}{3}} \Rightarrow u_1 = +1 + \sqrt{\frac{16}{3}} \approx 3{,}31$$

$$u_2 = 1 - \sqrt{\frac{16}{3}} \approx -1{,}31 \notin J$$

$$\underline{\underline{u = 3{,}31}}$$

$$d''(u) = -\frac{3}{4}u + \frac{3}{4}$$

$$d''(3{,}31) = -1{,}732 < 0 \Rightarrow \text{Max}$$

<u>für $u = 3{,}31$ wird die Strecke am größten</u>

Punktverteilung

P5		W3a				W3b	
WT	0,5 P	t	1 P	g	1 P	d(u)	1,5 P
beiden	1,5 P	t_N	1,5 P	φ	0,5 P	d'(u)	0,5 P
f'	1,0 P	S	0,5 P			d''(u)	0,5 P
f''	0,5 P	R	0,5 P			u	0,5 P
HP/TP	4 P	A_1	0,5 P			Max	0,5 P
		A_2	0,5 P				
		%	0,5 P				