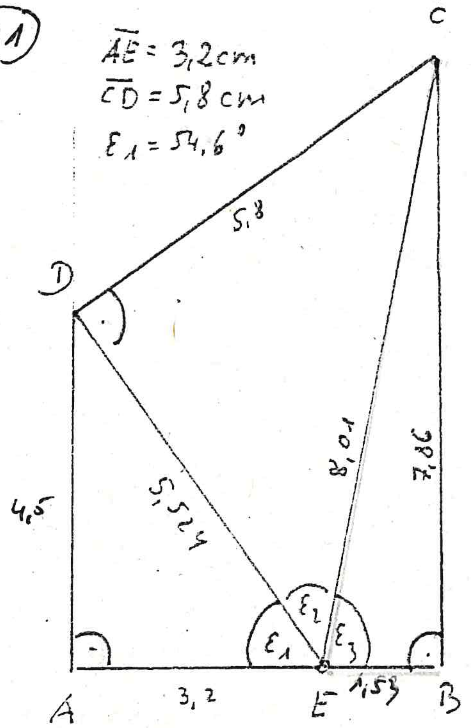
Q34 Waldorf HT 2014 P1/P2

(P1)

$\overline{AE} = 3{,}2\,cm$
$\overline{CD} = 5{,}8\,cm$
$\varepsilon_1 = 54{,}6°$



ΔAED: $\cos \varepsilon_1 = \dfrac{\overline{AE}}{\overline{DE}}$

$\Rightarrow \overline{DE} = 3{,}2 : \cos 54{,}6 = 5{,}524\,cm \rightarrow [A]$

ΔCDE: Pyth.: $\overline{CE} = \sqrt{\overline{DE}^2 + \overline{CD}^2} = 8{,}0092\,cm \rightarrow [B]$

$\tan \varepsilon_2 = \dfrac{\overline{CD}}{\overline{DE}} = \dfrac{5{,}8}{5{,}524} = 1{,}05 \Rightarrow \varepsilon_2 = 46{,}4° \rightarrow [C]$

ΔBCE: $\varepsilon_3 = 180° - \varepsilon_1 - \varepsilon_2 = 79° \rightarrow [C]$

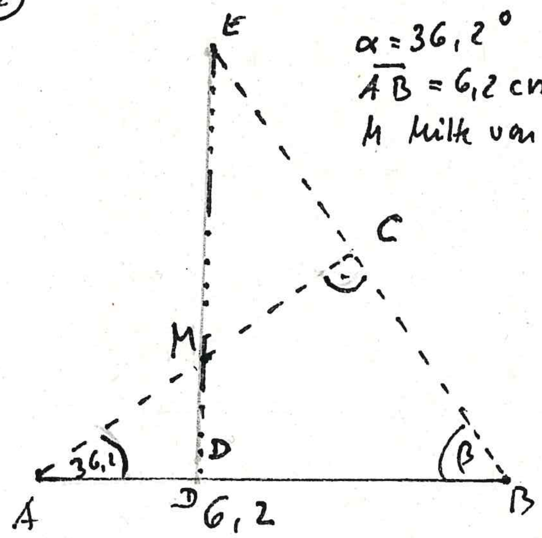
$\sin \varepsilon_3 = \dfrac{\overline{BC}}{\overline{CE}} \Rightarrow \overline{BC} = 8{,}01 \cdot \sin 79° = 7{,}86\,cm \rightarrow [D]$

Pyth.: $\overline{BE} = \sqrt{\overline{CE}^2 - \overline{BC}^2} = 1{,}53\,cm$

$\Rightarrow \underline{\underline{U_{\Delta EBC} = 17{,}4\,cm}} = 8{,}01 + 7{,}86 + 1{,}53$

(P2)

$\alpha = 36{,}2°$
$\overline{AB} = 6{,}2\,cm$
M Mitte von $\overline{AC}$



Für andere Wege:

$\overline{AD} = 2{,}04$ $\quad \overline{BC} = 3{,}67$
$\overline{DM} = 1{,}49$ $\quad \overline{EM} = 4{,}23$

$\underline{\Delta ABC:}$

$\cos \alpha = \dfrac{\overline{AC}}{\overline{AB}} \Rightarrow$

$\overline{AC} = 6{,}2 \cdot \cos 36{,}2° = 5{,}003 \rightarrow [x]$

$\overline{AM} = \frac{1}{2} \overline{AC} = 2{,}502 \rightarrow [y]$

$\underline{\Delta ADM:}$

$\cos \alpha = \dfrac{\overline{AD}}{\overline{AM}} \Rightarrow$

$\overline{AD} = 2{,}5 \cdot \cos 36{,}2° = 2{,}01867 \rightarrow [z]$

$\underline{\Delta BED:}$

$\overline{BD} = \overline{AB} - \overline{AD} = 4{,}181 \rightarrow [t]$

$\beta = 90° - 36{,}2° = 53{,}8°$

$\tan \beta = \dfrac{\overline{DE}}{\overline{BD}} \Rightarrow$

$\overline{DE} = 4{,}181 \cdot \tan 53{,}8° = 5{,}713$

$\Rightarrow \underline{\underline{\overline{DE} = 5{,}71\,cm}}$

RSA Waldorf HT 2014 P_3, P_4 W2a,b

P3 $\frac{x}{x+4} = \frac{3x+28}{x^2+4x} + \frac{1}{x} \quad | \cdot HN$

1\. Nenner: $x+4 \Rightarrow x \neq -4$

2\. Nenner: $x^2+4x = x(x+4) \Rightarrow x \neq 0,\ x \neq -4$

3\. Nenner: $x \Rightarrow x = 0$

$\Rightarrow \mathbb{D} = \mathbb{R} \setminus \{-4; 0\}$ $\quad HN = x(x+4) = x^2+4x$

$\frac{x \cdot x \cdot \cancel{(x+4)}}{\cancel{(x+4)}} = \frac{(3x+28)\cancel{(x^2+4x)}}{\cancel{(x^2+4x)}} + \frac{1 \cdot \cancel{x}(x+4)}{\cancel{x}}$

$x^2 = 3x+28 + x+4$

$x^2 - 4x - 32 = 0 \Rightarrow x_{1/2} = 2 \pm \sqrt{4+32} = 2 \pm 6$

$x_1 = 2-6 = -4 \notin \mathbb{D}$

$x_2 = 2+6 = 8 \in \mathbb{D} \Rightarrow \mathbb{L} = \{8\}$

P4 $N_1(-2|0) \Rightarrow x_1 = -2 \Rightarrow x+2 = 0$

$N_2(4|0) \Rightarrow x_2 = 4 \Rightarrow x-4 = 0 \Rightarrow y = (x+2)(x-4) \Rightarrow \underline{p: y = x^2-2x-8}$

$g: y = -2(x-2{,}5) - 4 \Rightarrow \underline{y = -2x+1}$

Schnitt: $x^2-2x-8 = -2x+1 \Rightarrow x^2 = 9 \Rightarrow x_1 = 3,\ x_2 = -3 \Rightarrow y_1 = -5,\ y_2 = +7$ $\quad \underline{S_1(3|-5)}$, $\underline{S_2(-3|7)}$

W2a p_1: über Scheitelpunkt: Vergleich zu $y = x^2$

$\Rightarrow S_1(-1|2) \Rightarrow y = (x+1)^2+2$

$\underline{p_1: y = x^2+2x+3}$

mittels LGS: $A(-2|3) \Rightarrow 3 = 4 - 2p + q$; $B(0|3) \Rightarrow 3 = +q$ $\Big\} \Rightarrow p = 2,\ \Rightarrow q = 3$

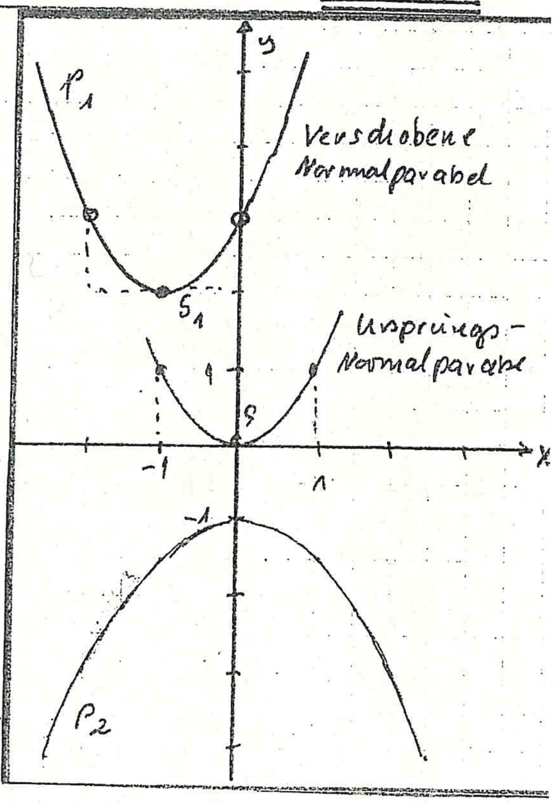
$\underline{p_1: y = x^2+2x+3}$

p_3: p muss flacher als p_1 sein z.B.: $\underline{y = \frac{1}{4}x^2}$ (oder flacher als p_2)

$\Rightarrow x^2+2x+3 = \frac{1}{4}x^2 \Rightarrow \frac{3}{4}x^2 + 2x + 3 = 0$

$x_{1/2} = \frac{-2 \pm \sqrt{4 - 4 \cdot \frac{3}{4} \cdot 3}}{2 \cdot 3/4}$ neg. unter Wurzel

$-\frac{1}{2}x^2 - 1 = \frac{1}{4}x^2 \Rightarrow \frac{3}{4}x^2 = -1 \Rightarrow x^2 = -\frac{4}{3}$ k. Lös.



W2b $A(-1|2)$: $2 = (-1)^2 - p - 1 \Rightarrow p = -2$

$2 = -1 + c \Rightarrow c = 3$

$\Rightarrow \underline{p_1: y = x^2-2x-1}$ $\quad \underline{p_2: y = -x^2+3}$

$\Rightarrow \underline{S_1(1|-2)}$ $\quad \underline{S_2(0|3)}$

Schnittpunkte:

$x^2-2x-1 = -x^2+3$

$2x^2-2x-4 = 0$

$x^2-x-2 = 0$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+2} \Rightarrow x_1 = -1 \Rightarrow A$; $x_2 = 2$, $y_2 = -1 \Big\} B(2|-1)$

$m_{S_1B} = \frac{-1-(-2)}{2-1} = 1$

$m_{S_2A} = \frac{2-3}{-1-0} = 1$ $\Big\}$ gleiche Steigung $\Rightarrow$ die Geraden sind parallel

Luca hat Recht.

W2b) $p_1: \quad y = x^2 + px - 1$ $\qquad$ $p_2: \quad y = -x^2 + c$

$A: \quad 2 = 1 - p - 1 \Rightarrow p = -2$ $\qquad$ $A: \quad 2 = -1 + c \Rightarrow c = 3$

$\underline{p_1: y = x^2 - 2x - 1}$ $\qquad$ $\underline{p_2: y = -x^2 + 3}$

$S_1 \quad y = (x-1)^2 - 1 - 1$ $\qquad$ $S_2(0|3)$

$S_1(1|-2)$

Schnittp. B: $\quad x^2 - 2x - 1 = -x^2 + 3 \Rightarrow 2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = \frac{1}{2} \pm \frac{3}{2}$

$x_1 = -1 \Rightarrow A(-1|2)$

$x = 2 \Rightarrow \underline{\underline{B(2|-1)}}$

$\underline{m_{1(SB)} = \frac{-1+2}{2-1} = 1}$ $\qquad$ $\underline{m_{2(S_2A)} = \frac{2-3}{-1-0} = 1}$ $\Rightarrow$ Parallelität

Luca hat Recht

RSA Walldorf HT 2014 P5/W3

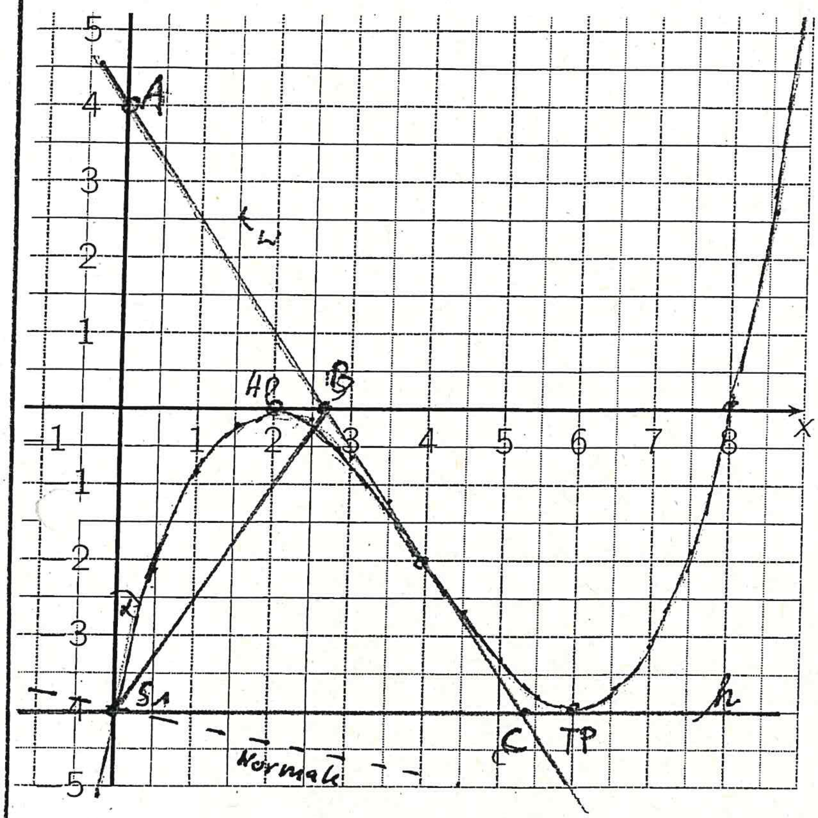
(P5) $f(x) = \frac{1}{8}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x - 4$ Abl. $f'(x) = \frac{3}{8}x^2 - 3x + \frac{9}{2}$

$f''(x) = \frac{3}{4}x - 3$

$f'''(x) = \frac{3}{4} \neq 0$ (hinr. Bed. für WP)

Wertetabelle:

X	0	1	2	3	4	5	6	7	8
Y	-4	-0,9	0	-0,6	-2	-3,4	-4	-3,1	0



EP: $f'(x) = 0$

$\frac{3}{8}(x^2 - 8x + 12) = 0$

$x_{1/2} = 4 \pm \sqrt{16-12} = 4 \pm 2$

$x_1 = 2$ $x_2 = 6$

$f''(2) = -\frac{3}{2} < 0$ $f''(6) = +\frac{3}{2} < 0$

$\Rightarrow$ HP(2|0) $\Rightarrow$ TP(6|-4)

Winkel

$f'(0) = \frac{9}{2} \Rightarrow \tan\varphi = \frac{9}{2} \Rightarrow$

$\varphi = 77{,}47°$

$\Rightarrow \alpha = 12{,}53°$ ist der Winkel

$m_n = -\frac{2}{9} \Rightarrow \tan\beta \Rightarrow \tan^{-1}|-\frac{2}{9}| = 12{,}53°$

$h: y = -4$ $\Rightarrow S_1(0|-4)$

(W3a) Schnitt h mit K_f $\frac{1}{8}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x - 4 = -4 \Rightarrow \frac{1}{8}(x^2 - 12x + 36) \cdot x = 0 \Rightarrow x_1 = 0$

$(x-6)^2 = 0 \Rightarrow x_2 = 6 \Rightarrow$ TP

Wendepunkt ist Mitte von HP, TP $\Rightarrow$ WP(4|-2)

oder $f''(x) = 0 \Rightarrow \frac{3}{4}x - 3 = 0 \Rightarrow x = 4$

Wendetangente: $f'(4) = -\frac{3}{2} \Rightarrow y = -\frac{3}{2}(x-4) - 2 \Rightarrow$ t_w: $y = -\frac{3}{2}x + 4$

$x = 0 \Rightarrow y = 4 \Rightarrow$ A(0|4)

Schnittp. B: $y = 0 \Rightarrow x = \frac{8}{3} \Rightarrow$ B($\frac{8}{3}$|0)

Schnittp. C: $-4 = -\frac{3}{2}x + 4 \Rightarrow x = \frac{16}{3} \Rightarrow$ C($\frac{16}{3}$|-4)

$A_{\Delta S_1BA} = \frac{1}{2} \cdot \overline{S_1A} \cdot x_B = \frac{1}{2} \cdot 8 \cdot \frac{8}{3} = \frac{32}{3}$ FE

$A_{\Delta S_1CB} = \frac{1}{2} \cdot x_C \cdot (y_B - y_{S_1}) = \frac{1}{2} \cdot \frac{16}{3} \cdot 4 = \frac{32}{3}$ FE

} Die Flächen sind gleich

oder argumentativ

(W3b) $f_a(x) = ax^3 - \frac{3}{2}x^2$

$f_a'(x) = 3ax^2 - 3x$

$f_a''(x) = 6ax - 3$

$f_a'''(x) = 6a \neq$

WP: $f_a''(x) = 0 \Rightarrow 6ax - 3 = 0$

$\Rightarrow x = \frac{1}{2a}$

$f_a\left(\frac{1}{2a}\right) = \frac{1}{8a^2} - \frac{3}{2} \cdot \frac{1}{4a^2} = -\frac{1}{4a^2}$

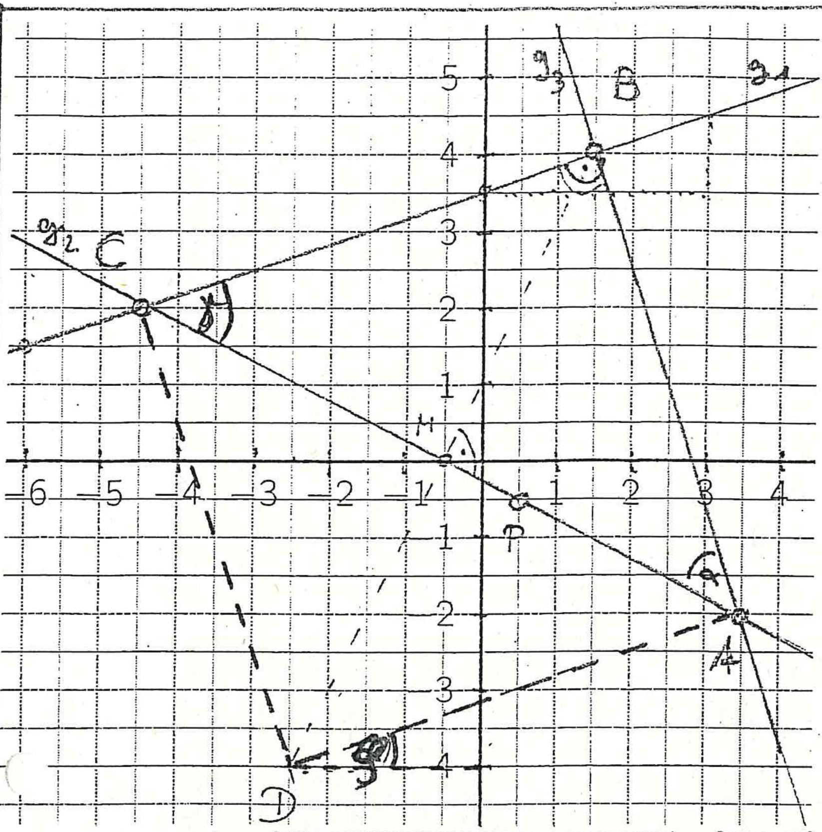
$\Rightarrow$ WP$\left(\frac{1}{2a} \middle| -\frac{1}{4a^2}\right)$

t_w: $f_a'\left(\frac{1}{2a}\right) = 3a \cdot \frac{1}{4a^2} - 3 \cdot \frac{1}{2a}$

$f_a'\left(\frac{1}{2a}\right) = -\frac{3}{4a}$

$y = -\frac{3}{4a}\left(x - \frac{1}{2a}\right) - \frac{1}{4a^2}$

t_w: $y = -\frac{3}{4a}x + \frac{1}{8a^2}$



P6 $g_1: y = \frac{1}{3}x + \frac{7}{2}$; $g_2: y = -\frac{1}{2}x - \frac{1}{4}$; $g_3: y = -3x + \frac{17}{2}$

$A(3{,}5|-2)$, $B(1{,}5|4)$, $C(-4{,}5|2)$, $P(0{,}5|-0{,}5)$

RSA Waldorf HT 2014 P6/W4

$g_2: y = \frac{2-(-0{,}5)}{-4{,}5-0{,}5}\cdot(x-(-4{,}5)) + 2$

$\underline{y = -\frac{1}{2}x - \frac{1}{4}}$

$g_3: \quad m = -\left(\frac{1}{\frac{1}{3}}\right) = -3$

$y = -3(x-3{,}5) - 2$

$\underline{y = -3x + 8{,}5 = -3x + \frac{17}{2}}$

Schnittpunkt B: $g_1 = g_3$

$\frac{1}{3}x + \frac{7}{2} = -3x + \frac{17}{2} \Rightarrow \frac{10}{3}x = \frac{10}{2}$

$\Rightarrow x = \frac{3}{2} \Rightarrow y = 4 \Rightarrow \underline{B\left(\frac{3}{2}\middle|4\right)}$

Punktprobe C auf g_1

$\underline{2} = \frac{1}{3}\cdot(-4{,}5) + \frac{7}{2} = \underline{2} \Rightarrow \underline{C \in g_1}$

Längen: $\overline{PC} = \sqrt{(-4{,}5-0{,}5)^2 + (2-(-0{,}5))^2} = \sqrt{31{,}25}$

$\overline{PA} = \sqrt{(3{,}5-0{,}5)^2 + (-2-(-0{,}5))^2} = \sqrt{11{,}25}$

$\Rightarrow \frac{\overline{PC}-\overline{PA}}{\overline{PC}}\cdot 100 = \underline{40\,\%} \Rightarrow$ $\underline{\overline{PA} \text{ ist um } 40\,\% \text{ kleiner als } \overline{PC}}$

W4a gleichschenklig: über Winkel: $\tan\gamma = \frac{1/3 - (-1/2)}{1 + \frac{1}{3}(-\frac{1}{2})} = 1 \Rightarrow \gamma = 45^\circ \Rightarrow \underline{\alpha = 45^\circ}$ da $\beta = 90^\circ$

über Längen: $\overline{BC} = \sqrt{(-4{,}5-1{,}5)^2 + (2-4)^2} = \sqrt{40}$; $\overline{AB} = \sqrt{(3{,}5-1{,}5)^2 + (-2-4)^2} = \sqrt{40}$ $\Big\}$ gleichschenklig

laut Text: ABCD bildet ein Quadrat $\Rightarrow$ $\underline{M \text{ ist Mitte von } \overline{AC}}$

$M\left(\frac{-4{,}5+3{,}5}{2}\middle|\frac{2-2}{2}\right) = \underline{M\left(-\frac{1}{2}\middle|0\right)}$ ($\Rightarrow$ Thales: Mittelpunkt des Kreises)

Mist Mitte von $\overline{BD}$ $\Rightarrow x_m = \frac{x_D + x_B}{2} \Rightarrow x_D = 2\cdot x_m - x_B = -1 - 1{,}5 = -2{,}5$

$y_m = \frac{y_D + y_B}{2} \Rightarrow y_D = 2y_m - y_B = 0 - 4 = -4 \Rightarrow \underline{D(-2{,}5|-4)}$

Drehwinkel: $m = \frac{1}{3} = \tan\varphi \Rightarrow \underline{\varphi \approx 18{,}43^\circ}$

W4b h_k: $Q(-\frac{1}{2}|0) = M$; $R(\frac{3}{2}|k) \Rightarrow y = \frac{k-0}{\frac{3}{2}+\frac{1}{2}}(x + \frac{1}{2}) + 0$

$\underline{y = \frac{k}{2}x + \frac{k}{4}}$

$g_2: y = -\frac{1}{2}x - \frac{1}{4}$

$h_k: y = \frac{k}{2}x + \frac{k}{4}$

Koeffizientenvergleich $\Rightarrow$

$\underline{k = -1}$

$(h_{-1} = g_2)$

$g_{\overline{BD}}: y = 2(x-1{,}5) + 4 = 2x + 1$

$h_k: y = \frac{k}{2}x + \frac{k}{4}$

Koeffizientenvergleich $\Rightarrow$ $\underline{k = 4}$

$(h_4 = g_{BD})$