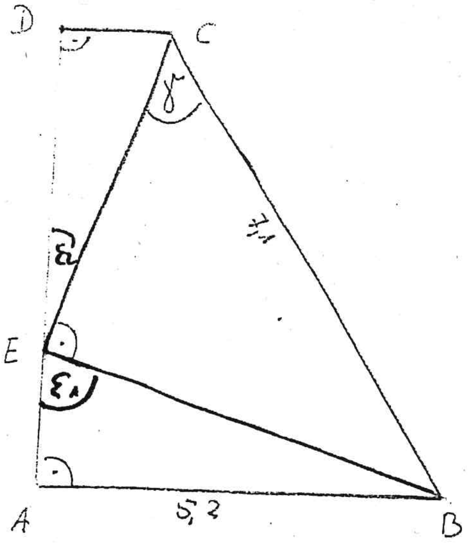


HT 2013
RSA
P1

ΔBCE

$\overline{BE}$: $\sin\gamma = \frac{\overline{BE}}{\overline{BC}}$

$\sin(50{,}5) = \frac{\overline{BE}}{7{,}1} \quad | \cdot 7{,}1$

$7{,}1 \cdot \sin(50{,}5) = \overline{BE} = 5{,}5\ cm \rightarrow [x]$.

$\overline{CE}$: $\cos\gamma = \frac{\overline{CE}}{\overline{BC}} \Rightarrow 7{,}1 \cdot \cos(50{,}5) = \overline{CE} = 4{,}5\ cm \rightarrow [y]$

oder Pythagoras

$\overline{BC}^2 = \overline{BE}^2 + \overline{CE}^2 \qquad 7{,}1^2 = 5{,}5^2 - \overline{CE}^2$

$\Rightarrow \overline{CE}^2 = 20{,}4 \Rightarrow \overline{CE} = 4{,}5\ cm$

ΔABE

$\overline{AE}$: Pythagoras: $\overline{AE}^2 + \overline{AB}^2 = \overline{BE}^2 \qquad \overline{AE}^2 + 5{,}2^2 = 5{,}5^2$

$\overline{AE}^2 = 5{,}5^2 - 5{,}2^2 = 2{,}97\ldots \Rightarrow \overline{AE} = 1{,}72\ldots \rightarrow [z]$

Winkel ε_1: $\tan(\varepsilon_1) = \frac{5{,}2}{\overline{AE}} = 3{,}015\ldots \Rightarrow \varepsilon_1 = 71{,}65^\circ$

ΔCDE

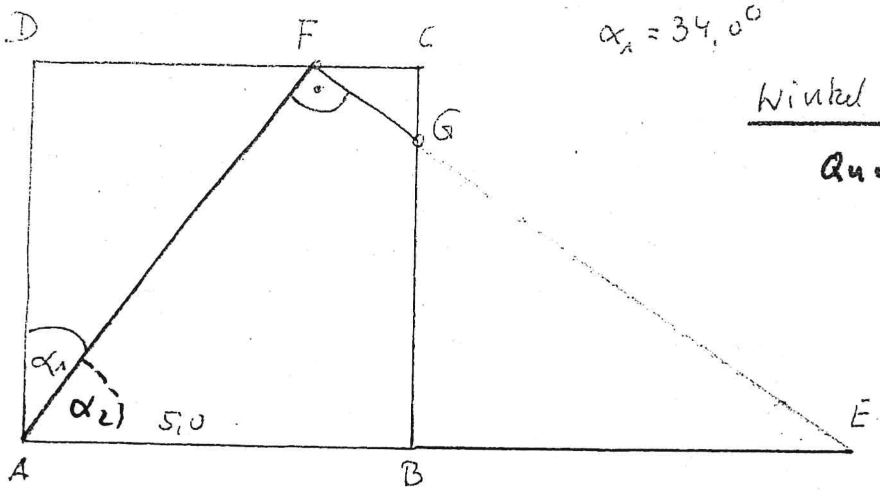
Winkel ε_2: $\varepsilon_2 = 180^\circ - 90^\circ - \varepsilon_1 = 18{,}348\ldots^\circ \rightarrow [t]$

$\overline{DE}$: $\cos(\varepsilon_2) = \frac{\overline{DE}}{\overline{CE}} \qquad \cos 18{,}35^\circ = \frac{\overline{DE}}{4{,}5} \Rightarrow$

$4{,}5 \cdot \cos 18{,}35^\circ = \overline{DE} = 4{,}286\ldots\ cm$

$\Rightarrow \overline{AD} = \overline{AE} + \overline{ED} = 1{,}72\ldots + 4{,}286\ldots = 6{,}01\ cm = \overline{AD}$

HT 2013
RSA
P2



$\alpha_1 = 34{,}0^0$

Winkel ε:

Quadrat: $\alpha_1 + \alpha_2 = 90^0 \Rightarrow \alpha_2 = 90^0 - 34^0$

$\Rightarrow \alpha_2 = 56^0$

Δ_{AEF}: $\alpha_2 + \varepsilon + 90^0 = 180^0$

$\Rightarrow \varepsilon = 180^0 - 90^0 - 56^0$

$\varepsilon = \alpha_1 = 34^0$

Δ_{AFD}: $\cos(\alpha_1) = \frac{\overline{AD}}{\overline{AF}}$ $\quad \cos(34) = \frac{5}{\overline{AF}} \Rightarrow \overline{AF} = \frac{5}{\cos(34)} = \underline{6{,}03\,cm} \to [x]$

$\overline{AF}$:

Δ_{AEF} $\quad \overline{AE}$: $\cos(\alpha_2) = \frac{\overline{AF}}{\overline{AE}}$ $\quad \cos(56^\circ) = \frac{6{,}03}{\overline{AE}} \Rightarrow$

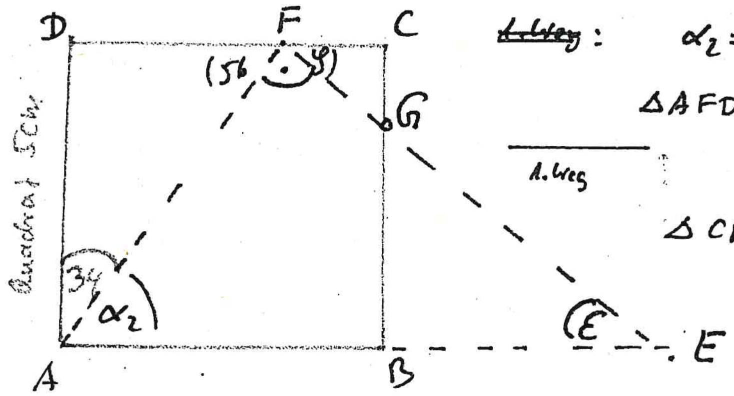
$\overline{AE} = \frac{6{,}03}{\cos(56)} = 10{,}785\ldots\,cm$

$\Rightarrow \overline{BE} = \overline{AE} - 5 = \underline{5{,}785\ldots\,cm} \to [y]$

Δ_{BEG} $\quad \cos(\varepsilon) = \frac{\overline{BE}}{\overline{EG}}$ $\quad \cos(34^0) = \frac{5{,}785}{\overline{EG}} \Rightarrow$

$\overline{EG} = \frac{5{,}785\ldots}{\cos(34)} = 6{,}978\ldots \Rightarrow \underline{\underline{\overline{EG} = 7\,cm}}$

P2



~~1. Weg~~: $\alpha_2 = 90^0 - 34^0 = 56^0 \Rightarrow \underline{\underline{\varepsilon = 90^0 - 56^0 = 34^0}}$

1. Weg

ΔAFD: $\tan 34^0 = \frac{\overline{DF}}{5} \Rightarrow \underline{\overline{DF} = 3{,}37\,cm}$

$\Rightarrow \overline{FC} = 5 - 3{,}37 = \underline{1{,}63\,cm = \overline{FC}}$

ΔCFG: $\varphi = \alpha_1 = 34^0$

$\tan 34^0 = \frac{\overline{CG}}{1{,}63} \Rightarrow \underline{\overline{CG} = 1{,}1\,cm}$

$\Rightarrow \overline{BG} = 5 - 1{,}1 = \underline{3{,}9\,cm = \overline{BG}}$

ΔBEG: $\sin 34^0 = \frac{3{,}9}{\overline{EG}} \Rightarrow \underline{\underline{\overline{EG} = 7\,cm}}$

2. Weg: $\overline{AF} = \sqrt{5^2 + 3{,}37^2 (\overline{DF})} = \underline{6{,}03\,cm = \overline{AF}}$

$\frac{\overline{EF}}{6{,}03} = \tan 34^0 \Rightarrow \underline{\overline{EF} = 8{,}9\,cm}$

$\underline{\overline{AE}} = \sqrt{6{,}03^2 + 8{,}9^2} = \underline{10{,}785\,cm} \Rightarrow \underline{\overline{BE}} = 10{,}785 - 5 = \underline{5{,}785\,cm}$

$\cos 34^0 (\varepsilon) = \frac{\overline{BE}}{\overline{EG}} \Rightarrow \underline{\underline{\overline{EG} = 7\,cm}}$

RSA 2013 HT P3, P4 W2a/b

(P3) $(3x+1)^2 + x(5-4x) = (\frac{1}{2}x-1)(6x+2) - 11$

$9x^2 + 6x + 1 + 5x - 4x^2 = 3x^2 + x - 6x - 2 - 11$

$2x^2 + 16x + 14 = 0$

$x^2 + 8x + 7 = 0 \Rightarrow x = -4 \pm \sqrt{16-7} = -4 \pm 3$

ABC oder Vieta

$x_1 = -1 \quad x_2 = -7$

(P4) $y = x^2 + 4x + q$ $A(-3|-4) \Rightarrow -4 = 9 - 12 + q$

$-1 = q \Rightarrow p: y = x^2 + 4x - 1$

$B(1|y_B) \in p \Rightarrow y_B = 1 + 4 - 1 = 4 \Rightarrow B(1|4)$

$S: y = (x+2)^2 - 4 - 1 = (x+2)^2 - 5 \Rightarrow S(-2|-5)$

oder $x_S =$
$y_S =$ Formelsammlung

Gerade durch S, B:

2 P Form

$\frac{y-4}{x-1} = \frac{-5-4}{-2-1} = 3$

$y = 3(x-1) + 4$

$g: y = 3x + 1$

2x Hauptform

$-5 = -2m + b$
$4 = m + b$

$-9 = -3m \quad m = 3$
$\Rightarrow b = 1$
$g: y = 3x + 1$

(W2a) $y = x^2 + px + q$

Tabelle / Zeichn. $Q(7|-4)$, $R(3|-4)$ $\Rightarrow$

$-4 = 49 + 7p + q$
$-4 = 9 + 3p + q$

$0 = 40 + 4p \Rightarrow p = -10 \Rightarrow q = 17$

$p_1: y = x^2 - 10x + 17$

Tabelle:

x	3	4	5	6	7	8	9
y	-4	-7	-8	-7	-4	1	8

$p_2: y = -x^2 - 4$ Schnittp.: $-x^2 - 4 = x^2 - 10x + 17$

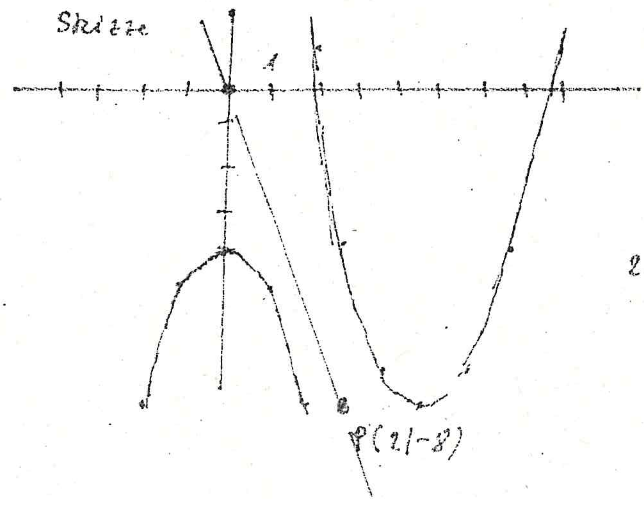
$0 = 2x^2 - 10x + 21$

$0 = x^2 - 5x + 10{,}5$

$x_{1/2} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{42}{4}}$ < 0

k. Lösung
kein Schn. p.

Skizze



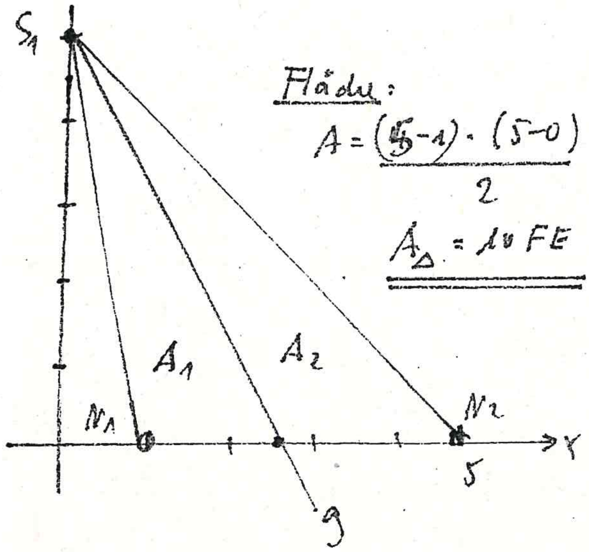
z.B. $y = -2x$
oder $-2x - 2$
oder $-2x - 1$

(W2 b) $P_1: \; y = -\frac{1}{2}x^2+5 \quad \Rightarrow S_1(0|+5)$

$P_2: \; S_2(3|-4)$

$y = (x-3)^2-4$

$p_2: \; y = x^2-6x+5 \quad \Rightarrow x = 3 \pm \sqrt{9-5}$ $\quad \underline{N_1(1|0)}$ $\quad \underline{N_2(5|0)}$



Fläche:

$A = \frac{(5-1)\cdot(5-0)}{2}$

$\underline{\underline{A_\Delta = 10 \text{ FE}}}$

Schnittpunkt:

$-\frac{1}{2}x^2+5 = x^2-6x+5$

$0 = \frac{3}{2}x^2-6x$

$0 = x\left(\frac{3}{2}x-6\right)$

$\Rightarrow x_1 = 0 \quad \Rightarrow y = 5 \quad S_1(0|5)$

$x_2 = 4 \quad \Rightarrow y = -3 \quad \underline{Sp(4|-3)}$

Gerade: $\frac{y-5}{x-0} = \frac{-3-5}{4-0} = -2$

$\underline{g: y = -2x+5}$

oder

S teilt $\overline{N_1N_2}$ (als Grundseite) nicht in der Mitte.

$A_1 = \frac{1}{2}\cdot\frac{3}{2}\cdot 5 = \frac{15}{4} = 3{,}75$ FE

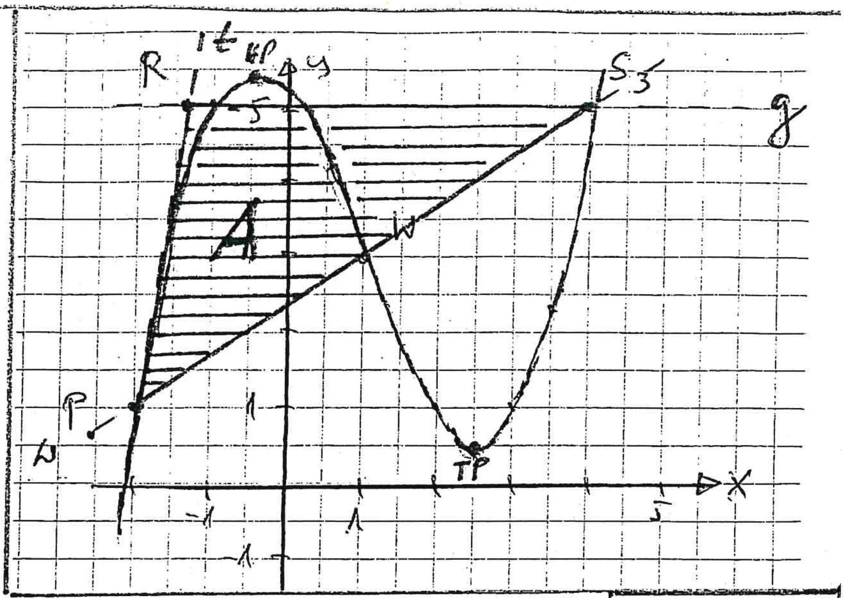
$A_2 = \frac{1}{2}\cdot\frac{5}{2}\cdot 5 = \frac{25}{4} = 6{,}25$ FE

(1) $A_1 \neq 5$ die Hälfte

(2) $A_1 \neq A_2 = 5$

P5

$f(x) = \frac{1}{3}x^3 - x^2 - \frac{4}{3}x + 5$



RSA 2013
HT
P5/W3a/b

Abl. $f'(x) = x^2 - 2x - \frac{4}{3}$

$f''(x) = 2x - 2$

$(f'''(x) = 2)$

Wertetabelle

x	-2	-1	0	1	2	3	4
$f(x)$	1	5	5	3	1	1	5

Hoch-/Tiefpunkte:

(notw. Bed.) $f'(x) = 0 \Rightarrow x^2 - 2x - \frac{4}{3} = 0$

$x_{1/2} = 1 \pm \sqrt{1 + \frac{4}{3}} = 1 \pm \sqrt{\frac{7}{3}}$

$\Rightarrow x_1 = 1 - \sqrt{\frac{7}{3}} = \frac{3 - \sqrt{21}}{3} \approx -0{,}53$

$x_2 = 1 + \sqrt{\frac{7}{3}} = \frac{3 + \sqrt{21}}{3} \approx 2{,}53$

(hinr. Bed.) $f''(x_1) = -\frac{2}{3}\sqrt{21} < 0 \Rightarrow HP$

$f(x_1) = 5{,}4 \Rightarrow \underline{\underline{HP(-0{,}5 | 5{,}4)}}$

$f''(x_2) = \frac{2}{3}\sqrt{21} > 0 \Rightarrow TP$

$f(x_2) = 0{,}6 \Rightarrow \underline{\underline{TP(2{,}5 | 0{,}6)}}$

W3 a

Gerade g: $y = 5$

Schnittpunkte g mit K_f:

$$\frac{1}{3}x^3 - x^2 - \frac{4}{3}x + 5 = 5 \quad | -5$$

$$\frac{1}{3}x^3 - x^2 - \frac{4}{3}x = 0$$

$$x\left(\frac{1}{3}x^2 - x - \frac{4}{3}\right) = 0 \Rightarrow x_1 = 0$$

Klammer mal 3

p-q: $x^2 - 3x - 4 = 0$

$$x_{2/3} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}} = \frac{3}{2} \pm \frac{5}{2}$$

$x_2 = -1$ und $x_3 = 4$

$\Rightarrow S_1(-1|5);\ S_2(0|5);\ S_3(4|5)$

Gerade w: Wendepunkt: $f''(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$

(hinr. Bed. $f'''(1) = 2 \neq 0$ ist nicht nötig)

$f(1) = 3 \Rightarrow WP(1|3)$

Version 2-Punkt-Form

$$\frac{y-3}{x-1} = \frac{5-3}{4-1} = \frac{2}{3}$$

$$y = \frac{2}{3}(x-1) + 3$$

w: $y = \frac{2}{3}x + \frac{7}{3}$

Version: 2 mal Punktprobe

$y = m \cdot x + b$

WP: $3 = m + b$
S_3: $5 = 4m + b$) $-$

$-2 = -3m \Rightarrow m = \frac{2}{3}$

einsetzen $\Rightarrow b = \frac{7}{3}$

w: $y = \frac{2}{3}x + \frac{7}{3}$

$P \in w$? Punktprobe: $1 = \frac{2}{3}(-2) + \frac{7}{3}$

$1 = -\frac{4}{3} + \frac{7}{3} = 1 \Rightarrow P \in w$

Tangente t in P $\quad P(-2|1)$ gegeben

$m_t = f'(-2) = \frac{20}{3} \Rightarrow$ Punkt-Steigungsform

$$y = \frac{20}{3}(x+2) + 1$$

$$t: y = \frac{20}{3}x + \frac{43}{3}$$

Schnittpunkt t mit g

$$5 = \frac{20}{3}x + \frac{43}{3}$$

$$-\frac{28}{3} = \frac{20}{3} \Rightarrow x = -\frac{7}{5} = -1{,}4 \Rightarrow \underline{\underline{R\left(-\frac{7}{5} \middle| 5\right)}}$$

$$\underline{\underline{R(-1{,}4 | 5)}}$$

Fläche Dreieck RPS_3:

Version ① $A = \frac{g \cdot h}{2}$

$g = \overline{S_3R} = x_{S_3} - x_r = 4 + 1{,}4 = 5{,}4$

$h = y_R - y_P = 5 - 1 = 4$

$$\underline{\underline{A}} = \frac{4 \cdot 5{,}4}{2} = \underline{\underline{10{,}8 \text{ FE}}}$$

Version ② Flächenformel

$$A = \frac{1}{2}\left[x_R(y_P - y_{S_3}) + x_P(y_S - y_R) + x_S(y_R - y_P)\right]$$

$$A = \frac{1}{2}\left(-1{,}4(1-5) - 2(5-5) + 4(5-1)\right]$$

$$\underline{\underline{A = 10{,}8 \text{ FE}}}$$

W3b Im $[-2; 1]$ verläuft K_f oberhalb w

d ist die Differenz der y-Werte an der Stelle $x = u$

$\Rightarrow \underline{d(u) = f(u) - w(u)}$ (oder $P \in w$: $P(u | w(u))$
$Q \in K_f$: $Q(u | f(u))$

$d(u) = \left(\frac{1}{3}u^3 - u^2 - \frac{4}{3}u + 5\right) - \left(\frac{2}{3}u + \frac{7}{3}\right)$ $d(u) = f(u) - w(u) = \overline{PQ}$

$$\underline{d(u) = \frac{1}{3}u^3 - u^2 - 2u + \frac{8}{3}}$$

Max: $d'(u) = u^2 - 2u - 2$

$d''(u) = 2u - 2$

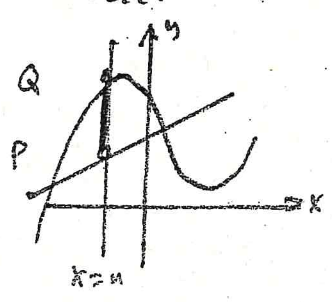
$d'(u) = 0 \Rightarrow u_{1/2} = 1 \pm \sqrt{3}$ $u_1 = 1 - \sqrt{3} \approx -0{,}732$
$u_2 = 1 + \sqrt{3} \approx 2{,}732 \notin [-2; 1]$

$\underline{\underline{u = 1 - \sqrt{3}}} \approx -0{,}732$

$d''(1 - \sqrt{3}) = -2\sqrt{3} < 0 \Rightarrow$ Max

$$\underline{\underline{d_{max} = d(1 - \sqrt{3}) = 2\sqrt{3} \approx 3{,}46}}$$

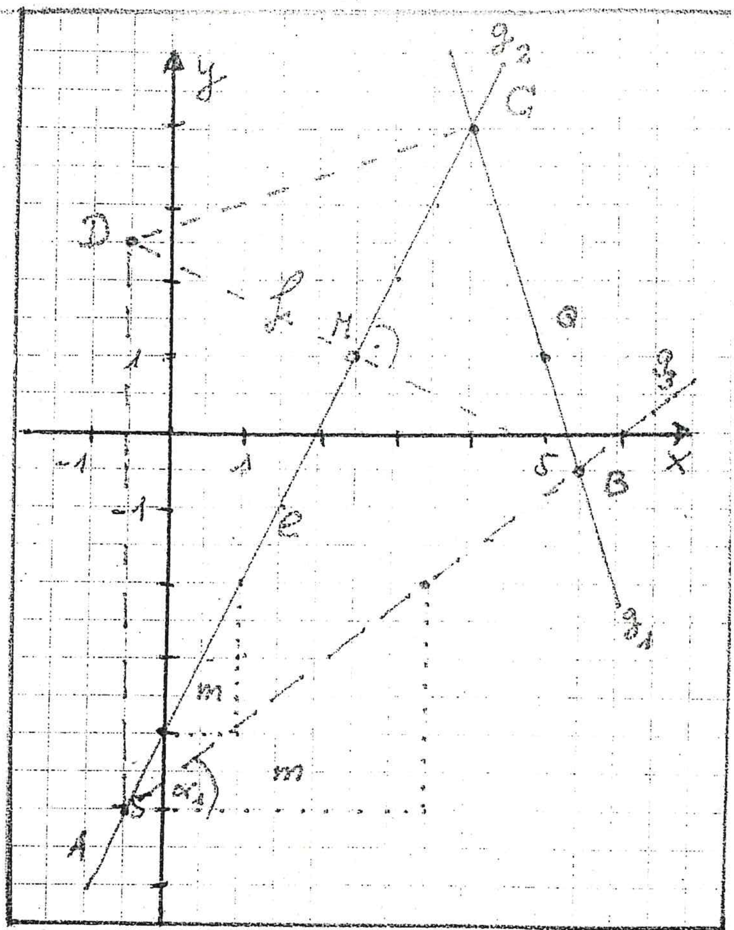
Skizze:



RSA 2013
HT
P6/W4 a/b

Zeichnung aus P6 und W3a

(P6)



(Steigungsdreieck nicht verlangt)

Gerade g_1: 2 Punktform

$$\frac{y-1}{x-5} = \frac{-0,5-1}{5,5-5,0} = -3$$

$$y = -3(x-5)+1$$

$$g_1:\ y = -3x+16$$

Gerade g_3: Punkt A, Steigung $\perp$ auf h $\Rightarrow$ m neg. rez.

$$m_3 = +\frac{3}{4}$$

$$\frac{y-(-5)}{x-(-0,5)} = \frac{3}{4}$$

$$y+5 = \frac{3}{4}\left(x+\frac{1}{2}\right)$$

$$g_3:\ y = \frac{3}{4}x - \frac{37}{8}$$

Schnittpunkt C: $-3x+16 = 2x-4$

$20 = 5x$

$x = 4 \Rightarrow y = 4$

$\Rightarrow C(4|4)$

A auf g_2? Punktprobe: $-5 = 2(-0,5)-4$

$-5 = -1-4$

$-5 = -5 \Rightarrow A \in g_2$

Unterschied: $\overline{AB} = \sqrt{(5,5-(-0,5))^2+(-0,5-(-5))^2} = \sqrt{56,25} = 7,5$ LE

$x_B = 5,5$ LE

$\frac{7,5-5,5}{5,5} \cdot 100\% = 36,\overline{36}\%$ Unterschied $36,4 \left(36\frac{4}{11}\right)\%$

(W4a) Nachweis Drachen (1) 2 Paar gleich langer Seiten $\overline{CD} = \overline{BC} = \sqrt{22,5}$ LE, $\overline{AD} = \overline{AB} = 7,5$ LE

oder (2) über $e \perp f$ und M (*)

oder (3) über Winkel

Nachweis Rechter Winkel $m_{DC} = \frac{1}{3}$ $m_{BC} = -3$ neg. rez. $\Rightarrow \gamma = 90°$

übrige Winkel

$\tan\alpha_1 = \frac{3}{4} \Rightarrow \alpha_1 = 36,87°$ -[A]

$\Rightarrow \alpha = 90° - \alpha_1 = 53,13°$ -[A]

$m_{DC} = \frac{1}{3} \Rightarrow \delta = 90° + \tan^{-1}\left(\frac{1}{3}\right) = 108,43°$

$\Rightarrow \beta = 360° - 90° - \alpha - \delta = 108,43°$

Fläche: Version ① Flächenformel für Dreieck

② über Diagonalen

③ über $A_\Delta = \frac{g \cdot h}{2}$

(*) $e = \sqrt{110{,}25}$
$f = \sqrt{45}$
$M(\frac{5}{2} | 1)$

Version C:

$g = \overline{AD} = 2{,}5 - (-5) = 7{,}5\ LE$
$h = x_C - x_D = 4 - (-0{,}5) = 4{,}5\ LE$

} Werte aus Zeichnung ablesbar!

$\Rightarrow A_{\Delta ACD} = \frac{1}{2} \cdot 7{,}5 \cdot 4{,}5 = 16{,}875\ FE\ (cm^2)$

$\Rightarrow \underline{\underline{A_{Drachen}}} = 2 \cdot 16{,}875 = \underline{\underline{33{,}75\ FE}}$

W4b $h_k: y = 2kx - 5k + 1$

$\underline{M(2{,}5|1) \in h_k?}$ Punktprobe: $2 \cdot k \cdot 2{,}5 - 5k + 1 = 1$

$5k - 5k + 1 = 1$

$\underline{\underline{0 = 0 \Rightarrow M \in h_k}}$

Schnittstelle x (1. Whb: $y = x$)

$x = 2kx - 5k + 1 \quad | -x + 5k - 1$

$5k - 1 = 2kx - x$

$= x(2k - 1) \quad | : (2k - 1)$ (Anm. $k \neq \frac{1}{2}$ nicht gefragt.)

$\underline{\underline{\frac{5k-1}{2k-1} = x}}$

k wenn $x = 4$

$\frac{5k-1}{2k-1} = 4 \quad | \cdot (2k - 1)$

$5k - 1 = 8k - 4$

$3 = 3k \Rightarrow \underline{\underline{k = 1}}$