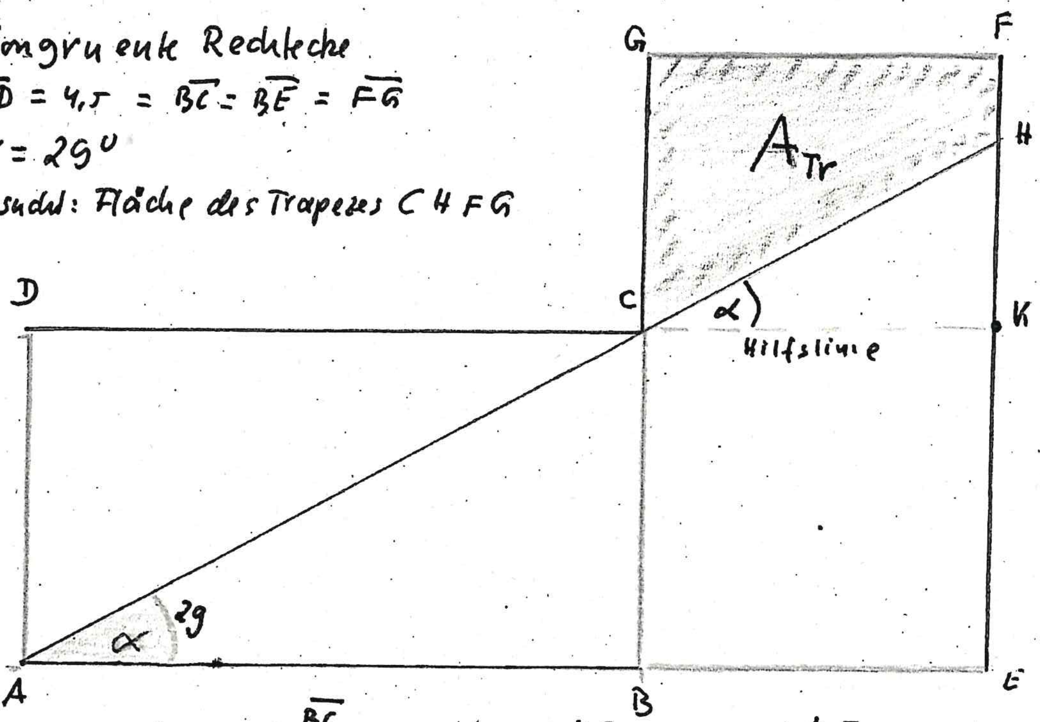
RAP
HT 2012
P1

2 Kongruente Rechtecke
$\overline{AD} = 4{,}5 = \overline{BC} = \overline{BE} = \overline{FG}$
$\alpha = 29^\circ$
gesucht: Fläche des Trapezes CHFG



$\triangle ABC$: $\tan\alpha = \dfrac{\overline{BC}}{\overline{AB}}$; $\tan(29) = \dfrac{4{,}5}{\overline{AB}} \Rightarrow \overline{AB} = \dfrac{4{,}5}{\tan(29)} = 8{,}12\,cm \to [x]$
$\Rightarrow \overline{AB} = \overline{CD} = \overline{BG} = \overline{EF} = 8{,}12\,cm$

Weg I: Trapez mittels $A = m \cdot h$
$\underline{\overline{CG}} = \overline{BG} - \overline{BC} = 8{,}12 - 4{,}5 = \underline{3{,}62\,cm} \to [y]$

$\triangle AEH$: $\overline{AE} = \overline{AB} + \overline{BE} = 8{,}12 + 4{,}5 = 12{,}62\,cm \to [z]$
$\tan(29) = \dfrac{\overline{EH}}{12{,}62} \Rightarrow \overline{EH} = 12{,}62 \cdot \tan(29) = 6{,}99\,cm\ [t]$
$\Rightarrow \overline{FH} = \overline{EF} - \overline{EH} = 8{,}12 - 6{,}99 = \underline{1{,}12\,cm} \to [a]$
Trapez: $\underline{\underline{A}} = \underbrace{\dfrac{\overline{CG} + \overline{FH}}{2}}_{m} \cdot \underbrace{\overline{GF}}_{h} = \dfrac{3{,}62 + 1{,}12}{2} \cdot 4{,}5 = 2{,}37 \cdot 4{,}5 = \underline{\underline{10{,}67\,cm^2}}$

Weg II: Fläche BEFG (Rechteck) − Fläche BEKC (Quadrat) − Fläche CKH (Dreieck) = Fläche CHFG (Trapez)

$\triangle CKH$: $\alpha = 29$ als Stufenwinkel
$\overline{KH}$: $\tan(29) = \dfrac{\overline{KH}}{4{,}5} \Rightarrow \overline{KH} = 4{,}5 \cdot \tan(29) = 2{,}49\,cm \to [y]$

$A_{Rechteck} = 4{,}5 \cdot 8{,}12\,cm^2 = 36{,}53\,cm^2 \to [z]$
$A_{Dreieck} = \frac{1}{2} \cdot 4{,}5 \cdot 2{,}49\,cm^2 = 5{,}61\,cm^2 \to [t]$
$A_{Quadrat} = 4{,}5^2 = 20{,}25\,cm^2 \to [a]$

$\underline{\underline{A_{Tr}}} = 36{,}53 - 5{,}61 - 20{,}25 = \underline{\underline{10{,}67\,cm^2}}$

RAP
HT 2012
P2

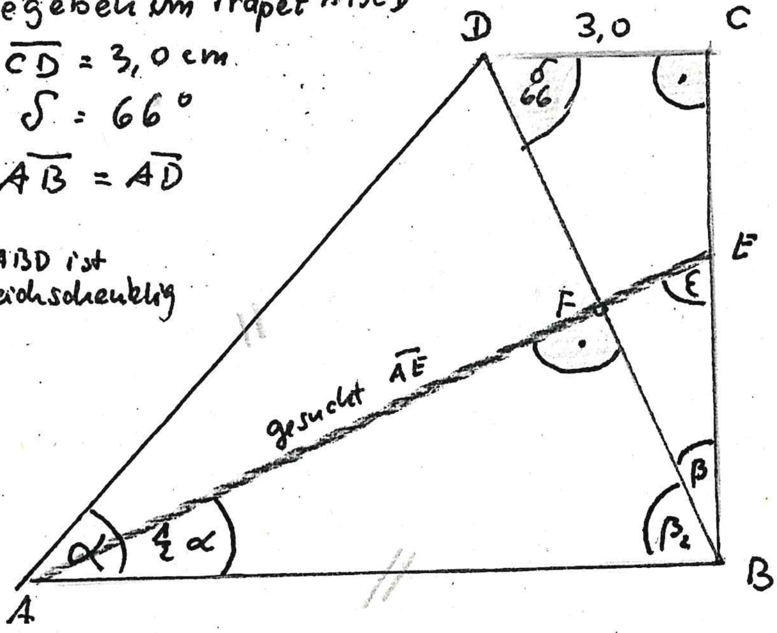
gegeben im Trapez ABCD

$\overline{CD} = 3{,}0$ cm

$\delta = 66°$

$\overline{AB} = \overline{AD}$

$\Rightarrow \Delta ABD$ ist gleichschenklig



$\underline{\Delta BCD}$

$\cos\delta = \frac{3{,}0}{\overline{BD}} \Rightarrow$

$\overline{BD} = \frac{3{,}0}{\cos 66} = 7{,}38$ cm → [x]

F ist Mitte von $\overline{BD}$ wegen Gleichschenkligkeit

$\Rightarrow \overline{FB} = \frac{1}{2}\overline{BD} = 3{,}69$ cm → [y]

$\underline{\Delta BEF:}$ $\beta = 90 - \delta = 24°$

$\varepsilon = \delta = 66°$

$\cos\beta = \frac{\overline{BF}}{\overline{BE}} \Rightarrow \overline{BE} = \frac{3{,}69}{\cos 24°} \Rightarrow$

$\overline{BE} = 4{,}04$ cm → [z]

$\underline{\Delta ABE:}$ $\cos\varepsilon = \cos\delta = \frac{\overline{BE}}{\overline{AE}} \Rightarrow \overline{AE} = \frac{3{,}69}{\cos(66)} = 9{,}925$

<u>Die Länge von $\overline{AE}$ beträgt 9,93 (9,9) cm</u>

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2 Weg: $\beta_2 = 66° \Rightarrow \alpha = 180° - 2 \cdot 66° = 48° \Rightarrow \frac{1}{2}\alpha = 24°$

ΔABF $\overline{AB}$ berechnen ($\overline{AB} = 9{,}07$ cm)

$\Delta ABE \Rightarrow \overline{AE}$

PSA
HT
2012
P1/P2
W1b

W1b $\triangle BCE$: $\cos 30^\circ = \frac{\overline{EB}}{6\cdot\sqrt{3}} \Rightarrow \underline{\underline{\overline{EB} = \frac{1}{2}\cdot\sqrt{3}\cdot 6\cdot\sqrt{3} = 9\,\text{cm}}}$

Pyth: $\overline{BE} = \sqrt{\overline{BC}^2 - \overline{BE}^2} = \sqrt{108e^2 - 81e^2} = \sqrt{27e^2} = \underline{\underline{3\cdot\sqrt{3}\,e = \overline{BE}}}$

$\triangle ABE$: $\beta = 30^\circ \quad \alpha = 60^\circ$

$\cos 30^\circ = \frac{\overline{BE}}{\overline{AB}} \Rightarrow \overline{AB} = \frac{3\cdot\sqrt{3}e}{\frac{1}{2}\sqrt{3}} = \underline{\underline{6e = \overline{AB}}}$

$\triangle ABD$: $\tan 30^\circ = \frac{\overline{AD}}{\overline{AB}} \Rightarrow \overline{AB}\cdot\tan 30^\circ = \overline{AD}$

$\overline{AD} = 6e\cdot\frac{1}{3}\sqrt{3} = \underline{\underline{2\sqrt{3}\,e = \overline{AD}}}$

Pyth: $\overline{BD} = \sqrt{\overline{AB}^2 + \overline{AD}^2} = \sqrt{(6e)^2 + (2\sqrt{3}e)^2}$

$= \sqrt{36e^2 + 12e^2} = \sqrt{48e^2} = \underline{4\cdot\sqrt{3}\,e = \overline{BD}}$

Strecke $\overline{DE} = \overline{BD} - \overline{BE} = (4\cdot\sqrt{3} - 3\sqrt{3})e = \sqrt{3}\,e$

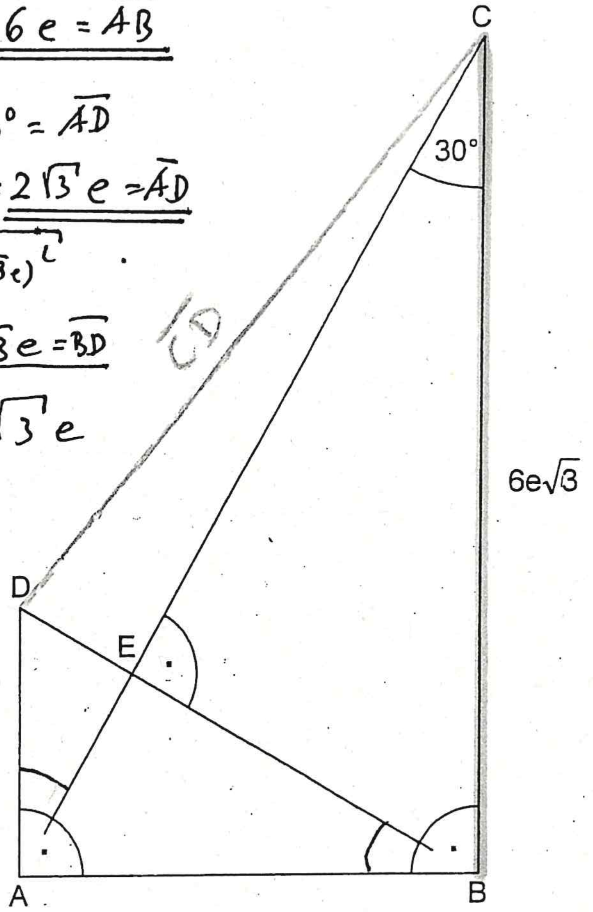
$\triangle DEC$:

$\overline{CD} = \sqrt{\overline{DE}^2 + \overline{CE}^2}$

$= \sqrt{3e^2 + 81e^2} = \sqrt{84e^2}$

$= \sqrt{4\cdot 21\cdot e^2}$

$\Rightarrow \boxed{\overline{CD} = 2\cdot\sqrt{21}\,e}$



W1a $P(\text{rot}) = 25\% = \frac{1}{4}$
$P(\text{gelb}) = \frac{1}{3}$ $\Rightarrow P(\text{blau}) = 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$

$\Rightarrow E(X) = 4\cdot\frac{1}{4} + 1{,}5\cdot\frac{1}{3} + 0{,}6\cdot\frac{5}{12} - 2 = -0{,}25$

Die Klasse gewinnt auf lange Sicht durchschnittlich 0,25 € pro Spiel.

Veränderung:

-1- : $E(X) = a\cdot\frac{1}{4} + 1{,}5\cdot\frac{1}{3} + 0{,}6\cdot\frac{5}{12} - 2 = -0{,}5$

$\Rightarrow \frac{1}{4}a = \frac{3}{4} \Rightarrow$ Gewinn für Rot nur 3 €

-2- $E(X) = 4\cdot\frac{1}{4} + b\cdot\frac{1}{3} + 0{,}5\cdot\frac{5}{12} - 2 = -0{,}5$

$\Rightarrow \frac{b}{3} = \frac{1}{4} \Rightarrow b = 0{,}75$ Gewinn für Gelb nur noch 0,75 €

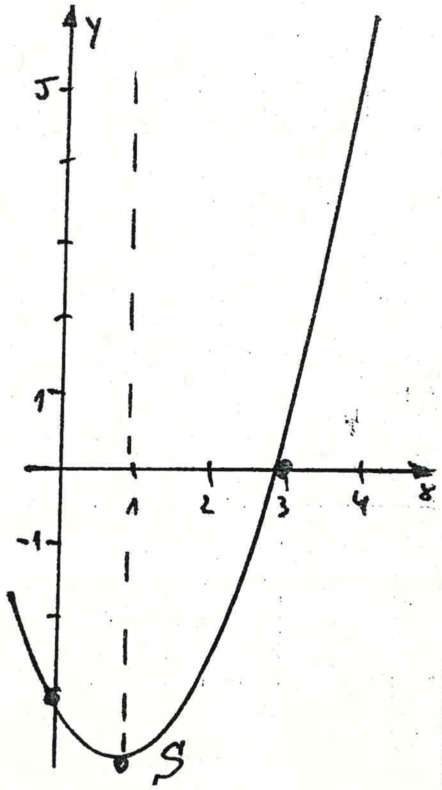
-3- $E(X) = 4\cdot\frac{1}{4} + 1{,}5\cdot\frac{1}{3} + c\cdot\frac{5}{12} - 2 = -0{,}5$

$\Rightarrow \frac{5}{12}c = 0$ kein Gewinn für blau

-4- $E(X) = (4-x)\cdot\frac{1}{4} + (1{,}5-x)\cdot\frac{1}{3} + (0{,}6-x)\cdot\frac{5}{12} - 2 = -0{,}5$

$\Rightarrow -x = -0{,}25$
$x = 0{,}25$ alle Gewinne werden um 0,25 € gekürzt

HT 2012
P3



Variante 1: über Scheitelpunkt S:

aus Zeichnung: $S(1|-4)$

$\Rightarrow$ Scheitelpunktform

$$y = (x-1)^2 - 4 = x^2 - 2x - 3$$

x-Achsenpunkte:

$y = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x_{1/2} = 1 \pm \sqrt{1+3} = 1 \pm 2$

$x_1 = -1 \Rightarrow \underline{\underline{N_1(-1|0)}}$

$x_2 = 3 \Rightarrow N_2(3|0)$ war gegeben

Variante 2 Parabel ist symmetrisch zur Gerade $x = 1$ (durch Scheitelpunkt)

$\Rightarrow x_1$ ist das Mittel von 1 und $x_2 = 3$

$\frac{x_1 + 3}{2} = 1 \Rightarrow x_1 = -1 \Rightarrow \underline{\underline{N_1(-1|0)}}$

$\Rightarrow \underline{\underline{y = (x+1)(x-3) = x^2 - 2x - 3}}$

Gerade g: $N_1(-1|0)$; $P(8|36)$ 2-Punkte-Form

$$y = \frac{36-0}{8-(-1)} \cdot (x-(-1)) + 0$$

$y = 4(x+1) \Rightarrow \underline{\underline{g: y = 4x + 4}}$

Schnittpunkt Q:

$x^2 - 2x - 3 = 4x + 4$

$x^2 - 6x - 7 = 0$

$x_{1;2} = 3 \pm \sqrt{9+7} = 3 \pm \sqrt{16} = 3 \pm 4$

$x_1 = -1 \Rightarrow N_1(-1|0)$

$\underline{\underline{x_2 = 7 \Rightarrow Q(7|32)}}$ $y_Q = 4 \cdot 7 + 4 = 28$

RSA HT 2012 P3 P4 W2

P3

I $2(x-3y)-(x-y)=7$

$2x-6y-x+y=7$

$x-5y=7$

II $2(5y-x)+16=\frac{4x-2}{3} \quad |\cdot 3$

$6(5y-x)+48=4x-2$

$30y-6x-4x=-50$

$-10x+30y=-50$

$\left.\begin{array}{l} x-5y=7 \\ -10x+30y=-50 \end{array}\right| \begin{array}{l}\cdot 6\\ \cdot 1\end{array} \Rightarrow -4x=-8 \Rightarrow x=2 \Rightarrow y=-1 \Rightarrow \mathbb{L}=\{(2|-1)\}$

P4 Argument (a) x_S ist Mittelwert von x_{N_1} und $x_{N_2} \Rightarrow x_{N_1}=-1 \Rightarrow N_1(-1|0)$

Rechnung: (b) $y=(x-1)^2-4$ Scheitelform

$(x-1)^2=4 \Rightarrow x-1=\pm 2$

$x=1\pm 2 \Rightarrow x_1=-1 \Rightarrow N_1(-1|0)$

$x_2=3 \Rightarrow N_2(3|0)$

Parabelgleichung

(a) $y=(x+1)(x-3)=x^2-2x-3$

(b) $y=x^2-2x+1-4=x^2-2x-3$

Gerade g: $y=\frac{36-0}{8+1}(x+1)+0 \Rightarrow y=4x+4$

$g\cap P=\{N_1; S\}$

$x^2-2x-3=4x+4$

$x^2-6x-7=0 \qquad x_{1/2}=+3\pm 4 \qquad x_1=-1 \Rightarrow N_1(-1|0)$

$x_2=7 \Rightarrow Q(7|32)$

W2a P_1: $y=-x^2+7{,}5$ (y-Achsensymm., nach unten geöffnet) $\Rightarrow S_1(0|\frac{15}{2})$

$g: y=-x+\frac{3}{2}$

$p_1\cap g=\{P; Q\} \qquad -x^2+\frac{15}{2}=-x+\frac{3}{2} \Rightarrow 0=x^2-x-6 \Rightarrow x_1=+3 \Rightarrow y=-\frac{3}{2}$

$x_2=-2 \Rightarrow y=+\frac{7}{2}$

$\Rightarrow P(-2|\frac{7}{2}); \; Q(3|-\frac{3}{2})$

Parabel: $y=x^2+px+q$

$\left.\begin{array}{ll} P\Rightarrow & -\frac{3}{2}=9+3p+q \\ Q\Rightarrow & \frac{7}{2}=4-2p+q \end{array}\right| \begin{array}{l}\cdot 1\\ \cdot(-1)\end{array} \Rightarrow -5=5+5p \Rightarrow p=-2$

$\Rightarrow q=-\frac{9}{2}$

P_2: $y=x^2-2x-\frac{9}{2}$

$y=(x-1)^2-1-\frac{9}{2}$

$\Rightarrow S_2(1|-\frac{11}{2})$

Viereck S_1PS_2Q

$m_1=m_{PS_2}=\frac{-\frac{11}{2}-\frac{7}{2}}{1+2}=-3 \qquad m_3=m_{PS_1}=\frac{\frac{15}{2}-\frac{7}{2}}{0+2}=2$

$m_2=m_{S_1Q}=\frac{-\frac{3}{2}-\frac{15}{2}}{2}=-3 \qquad m_4=m_{S_2Q}=\frac{-\frac{3}{2}+\frac{11}{2}}{2}=2$

2 Paare paralleler Seiten $\Rightarrow$ Parallelogramm

(Lösungsweg auch über Strecken oder Winkel)

W 2 b) $P(3|12)$ Symmetrieachse durch $A(-1|0)$

$x_{Scheitel}: x = -1$

$\Rightarrow y = (x+1)^2 + y_S$

$P \Rightarrow 12 = (3+1)^2 + y_S \Rightarrow y_S = -4$

$\Bigg\} \quad p: y = (x+1)^2 - 4$

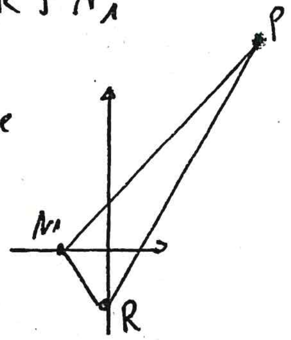
$\underline{\underline{y = x^2 + 2x - 3}}$

$N_1; N_2 \qquad x_{1/2} = -1 \pm 2 \Rightarrow \underline{\underline{N_1(-3|0);\ N_2(1|0)}}$

$R: \qquad x_R = 0 \Rightarrow y_R = -3 \qquad \underline{\underline{R(0|-3)}}$

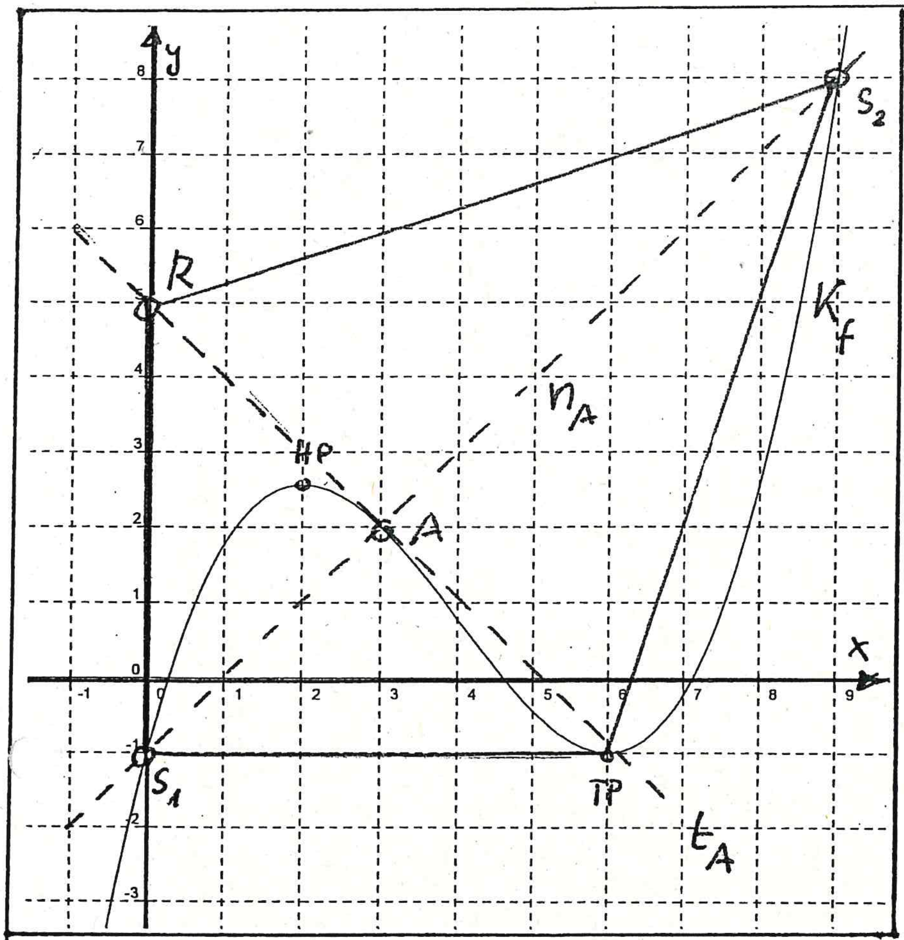
Dreieck RPN_1

Skizze



$A = \frac{1}{2}\Big[(0-12) + 3(0+3) + (-3)(-3-12)\Big]$

$A = \frac{1}{2}(0 + 9 + 45) = \frac{1}{2} \cdot 54 = \underline{\underline{27\,FE = A_\Delta}}$



P5

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P5
W3

$f(x) = \frac{1}{9}x^3 - \frac{4}{3}x^2 + 4x - 1$

$f'(x) = \frac{1}{3}x^2 - \frac{8}{3}x + 4$

$f''(x) = \frac{2}{3}x - \frac{8}{3}$

Tabelle:

x	0	1	2	3	4	5	6	7
f(x)	-1	$\frac{16}{9}$ 1,8	$\frac{23}{9}$ 2,6	2	$\frac{7}{9}$ 0,8	$-\frac{4}{9}$ -0,4	-1	$-\frac{2}{9}$ -0,2

x	8	9
f(x)	$\frac{23}{9}$ 2,6	8

Extrempunkte:

n.B. $f'(x) = 0 = \frac{1}{3}x^2 - \frac{8}{3}x + 4 \mid \cdot 3$

$x^2 - 8x + 12 = 0 \quad x_{1/2} = 4 \pm 2$

$x_1 = 2 \quad x_2 = 6$

h.B. $f''(2) = -\frac{4}{3} < 0 \Rightarrow \underline{\underline{HP(2|\tfrac{23}{9})}}$

$f''(6) = +\frac{4}{3} > 0 \Rightarrow \underline{\underline{TP(6|-1)}}$

W3a

Normale in A (3|2): $f'(3) = -1 \Rightarrow m_n = +1 \Rightarrow y = 1(x-3)+2$

$\Rightarrow \underline{\underline{n_A: y = x - 1}}$

Schnitt mit K_f: $x - 1 = \frac{1}{9}x^3 - \frac{4}{3}x^2 - 4x - 1 \Rightarrow \frac{1}{9}x^3 - \frac{4}{3}x^2 - 3x = 0 \mid \frac{1}{9}x$ ausklam.

$\frac{1}{9}x(x^2 - 12x - 27) = 0 \Rightarrow x_1 = 0 \Rightarrow \underline{\underline{S_1(0|-1)}}$

$x^2 - 12x - 27 = 0 \Rightarrow x_{2/3} = 6 \pm 3 \quad x_2 = 3 \Rightarrow A(3|2)$

$x_3 = 9 \Rightarrow \underline{\underline{S_2(3|8)}}$

Tangente t in A: $y = -1(x-3)+2 \Rightarrow \underline{\underline{t_A: y = -x + 5}}$

Punkt R: $x = 0 \Rightarrow y = 5 \Rightarrow \underline{\underline{R(0|5)}}$

Begr. Drachen: 2 Paare gleich langer Seiten:

(i) $\overline{S_1TP} = 6\,LE = \overline{S_1R}$

(ii) $\overline{RS_2} = \sqrt{9^2 + 3^2} = \sqrt{90} \quad \overline{TP\,S_2} = \sqrt{3^2 + 9^2} = \sqrt{90}$

zudem $e \perp f$ (Diagonalen)

und A ist Mitte von $\overline{RTP}$

Fläche Drachen:

$$\left.\begin{array}{l} e = \overline{S_1S_2} = \sqrt{9^2+9^2} = 9\cdot\sqrt{2} \\ f = \overline{RTP} = \sqrt{6^2+6^2} = 6\cdot\sqrt{2} \end{array}\right\} \Rightarrow \underline{\underline{A_{Dr}}} = \frac{e\cdot f}{2} = \frac{9\cdot 6\cdot 2}{2} = \underline{\underline{54\ FE}}$$

Fläche Dreieck RS_1TP

$$A_1 = \frac{1}{2}\cdot 6\cdot 6 = 18\ FE \Rightarrow A_2 = A_{\Delta TPS_2R} = 36\ FE$$

Prozent: A_2 ist doppelt so groß wie A_1

also ist A_2 um 100% größer als A_1

(W3b)

$$g_a: y = \frac{1}{9}x^3 - ax^2$$

$$y' = \frac{1}{3}x^2 - 2ax$$

$$y'' = \frac{2}{3}x - 2a$$

$$y''' = \frac{2}{3} \neq 0 \quad (\text{Ex. von WP})$$

$$y'' = 0 \Rightarrow \frac{2}{3}x - 2a = 0$$

$$\frac{2}{3}x = 2a$$

$$\underline{x = 3a}$$

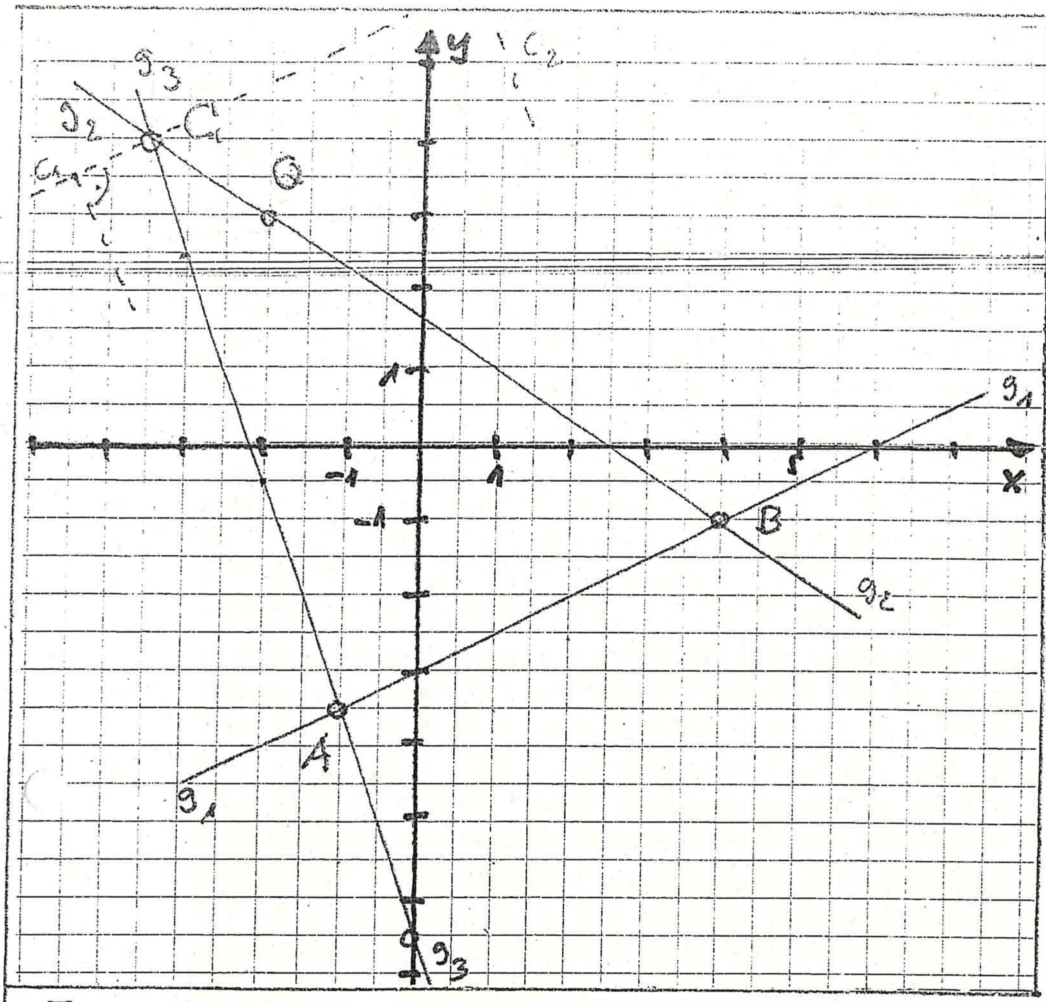
$$y = \frac{1}{9}\cdot 27a^3 - 9a^2 = -6a^3$$

$$WP_a(-3a \mid -6a^3)$$

$$y' = \frac{1}{3}\cdot 9a^2 - 2a\cdot 3a = -3a^2$$

$$\Rightarrow t: y = -3a^2(x-3a) - 6a^3$$

$$\underline{\underline{t_w: y = -3a^2x + 3a^3 = -3a^2(\cancel{x} - a)}}$$



(P6) g_1: (P-St-F)

$y = \frac{1}{2}(x+1) - 3{,}5$

$\underline{g_1: y = \frac{1}{2}x - 3}$

RSA HT 2011 P6/W4

g_2: Q; B (2PF)

$y = \left(\frac{-1-3}{4+2}\right)(x-4) - 1$

$\underline{y = -\frac{2}{3}x + \frac{5}{3} : g_2}$

g_3 gegeben

$\underline{y = -3x - 6{,}5}$

$B \in g_1$ Punktprobe

$-1 = \frac{1}{2} \cdot 4 - 3$

$-1 = -1 \Rightarrow \underline{B \in g_1}$

$g_2 \cap g_3 = \{C\}$

$-\frac{2}{3}x + \frac{5}{3} = -3x - 6{,}5 \Rightarrow$

$\frac{7}{3}x = -\frac{49}{6} \Rightarrow x = -3{,}5 \Rightarrow y = 4 \Rightarrow \underline{C_1(-3{,}5|4)}$

$d_Q = |y_Q| = 3$ $\quad d_A = |y_A| = 3{,}5$ $\quad \frac{3{,}5-3}{3{,}5} \cdot 100 = 14{,}2857\ldots$

$\Rightarrow$ Q ist um $\underline{14{,}3\%}$ näher an der x-Achse als A.

C bewegt sich auf g ∥ g_1 (AB) (W4a)

$\rightarrow$ P-St-F: $y = \frac{1}{2}(x+3{,}5) + 4 \Rightarrow \underline{g: y = \frac{1}{2}x + \frac{23}{4}}$

$\Delta_{ABC'}$ rechtwinklig: $\Rightarrow g_{AC'}$ hat Steigung $-2 \Rightarrow y = -2(x+1) - 3{,}5$ $\quad y = -2x - 5{,}5$

mit g schneiden $\Rightarrow -2x - 5{,}5 = \frac{1}{2}x + \frac{23}{4} \Rightarrow x = -4{,}5$

$\Rightarrow y = 3{,}5 \Rightarrow \underline{C_1(-4{,}5|3{,}5)}$

oder $\Rightarrow g_{BC}$ mit $m = -2 \Rightarrow y = -2(x-4) - 1 \Rightarrow y = -2x + 7$

mit g schneiden $\Rightarrow -2x + 7{,}0 = \frac{1}{2}x + \frac{23}{4} \Rightarrow x = 0{,}5$

$\Rightarrow y = 6 \Rightarrow \underline{C_2(0{,}5|6)}$

$A_{\Delta ABC_2} = \frac{1}{2}\left(-1(-1-6) + 4(6+3{,}5) + \frac{1}{2}(-3{,}5+1)\right) = \underline{21\frac{7}{8} \approx 21{,}9\,\text{FE} = A}$

h_k durch $R(4|0)$; $T_k(k|6)$ (W4b)

$$y = \frac{6-0}{k-4}(x-k)+6$$

einfacher

$$y = \frac{0-6}{4-k}(x-4)+0$$

$$y = \frac{-6}{4-k}(x-4) = \frac{6}{k-4}(x-4)$$

$$y = \frac{6}{k-4}x \quad \frac{-6k}{k-4}+6$$

geschickt: Absolutglieder zusammenfassen.

$$h_k: y = \frac{6}{k-4}x + \frac{\cancel{-6k} \cancel{+6k} -24}{k-4}$$

$$\underline{\underline{y = \frac{6}{k-4}\cdot(x-4)}}$$

$m_{g_1} = \frac{1}{2} \Rightarrow m_{h_k} = -2 = \frac{6}{k-4} \Rightarrow -2k+8=6 \Rightarrow \underline{\underline{k=1}}$