Lösungen Bruchgleichungen (11)

a) $\frac{5x^2+4x-23}{(x-1)(x+1)} = \frac{x-2}{x-1} + \frac{x+3}{x+1}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{1;-1\}$ $\quad | \cdot (x-1)(x+1)$

$5x^2+4x-23 = (x-2)(x+1) + (x+3)\cdot(x-1)$

$5x^2+4x-23 = x^2+x-2x-2 + x^2-x+3x-3$

$5x^2+4x-23 = 2x^2+x-5$ $\quad | -2x-x+5$

$3x^2+3x-18 = 0$ $\quad | :3$

$x^2+x-6 = 0$

p-q-Formel $\Rightarrow$ $x_1 = -3$ $x_2 = 2$ $\quad \mathbb{L} = \{-3;2\}$

$x_{1/2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+6}$

$\frac{6x^2+4x-23 \cdot (x+1)(x-1)}{(x-1)(x+1)} = \frac{(x-2)\cdot(x-1)(x+1)}{x-1} + \frac{(x+3)(x-1)(x+1)}{x+1}$

b) $\frac{x}{x-1} = \frac{3x-1}{x^2-1} - \frac{2x}{x+1}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{1;-1\}$ $\quad | \cdot (x+1)\cdot(x-1)$

NR: $(x-1)(x+1) = x^2-1$ [3. Binomische Formel]

$x\cdot(x+1) = 3x-1 - 2x\cdot(x-1)$

$x^2+x = 3x-1-2x^2+2x$

$x^2+x = -2x^2+5x-1$ $\quad | +2x^2-5x+1$

$3x^2-4x+1 = 0$

abc-Formel: $x_{1/2} = \frac{+4 \pm \sqrt{16-4\cdot3\cdot1}}{2\cdot3} = \frac{4\pm\sqrt{16-12}}{6} = \frac{4\pm\sqrt{4}}{6}$

$x_1 = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}$

$x_2 = \frac{4+2}{6} = \frac{6}{6} = 1$

$\mathbb{L} = \{\frac{1}{3}\}$ da $1 \notin \mathbb{D}$

c) $2 = \frac{5}{x^2+x} + \frac{x+6}{x+1}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{-1\}$ $\quad | \cdot x\cdot(x+1)$

NR: $x^2+1x = x\cdot(x+1)$

$2\cdot x(x+1) = 5 + (x+6)\cdot x$

$2x^2+2x = 5 + x^2+6x$

$2x^2+2x = x^2+6x+5$ $\quad | -x^2-6x-5$

$x^2-4x-5 = 0$

$x_{1/2} = 2 \pm \sqrt{4+5} \Rightarrow x_1 = -1 \quad x_2 = 5$

$\mathbb{L} = \{5\}$ da $-1 \notin \mathbb{D}$

$2\cdot x\cdot(x+1) = \frac{5\cdot x\cdot(x+1)}{x^2+x} + \frac{(x+6)\cdot x\cdot(x+1)}{x+1}$

d) $\frac{x-1}{x+2} = \frac{x+3}{2x} - 1$ $\quad \mathbb{D} = \mathbb{R} \setminus \{-2; 0\}$ $\quad | \cdot (x+2) \cdot 2x$

(*) $(x-1) \cdot 2x = (x+3) \cdot (x+2) - 1 \cdot (x+2) \cdot 2x$

$2x^2 - 2x = x^2 + 2x + 3x + 6 - 2x^2 - 4x$ [zusammenfassen]

$2x^2 - 2x = -x^2 + 1x + 6$ $\quad | + x^2 - 1x - 6$

$3x^2 - 3x - 6 = 0$ $\quad | : 3$

$x^2 - x - 2 = 0$

$x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = \frac{1}{2} \pm \sqrt{\frac{9}{4}}$

$x_1 = \frac{1}{2} - \frac{3}{2} = -1$ $\quad \mathbb{L} = \{-1; 2\}$

$x_2 = \frac{1}{2} + \frac{3}{2} = 2$

(**) $\frac{(x-1) \cdot \cancel{(x+2)} \cdot 2x}{\cancel{x+2}} = \frac{(x+3)\,\cancel{(x+2)} \cdot \cancel{2x}}{\cancel{2x}} - 1 \cdot (x+2) \cdot 2x$

e) $\frac{8x^2 + 25x - 20}{2x^2 - 2x} = \frac{2x-1}{2x-2} + \frac{x+5}{x}$ $\quad \mathbb{D} = \mathbb{R} \setminus \{0; 1\}$ $\quad | \cdot x \cdot (2x-2)$ $\quad$ NR: $2x^2 - 2x = x \cdot (2x-2)$

$8x^2 + 25x - 20 = (2x-1) \cdot x + (x+5) \cdot (2x-2)$

$8x^2 + 25x - 20 = 2x^2 - x + 2x^2 - 2x + 10x - 10$

$8x^2 + 25x - 20 = 4x^2 + 7x - 10$ $\quad | - 4x^2 - 7x + 10$

$4x^2 + 18x - 10 = 0$ $\quad | : 4$

$x^2 + \frac{18}{4}x - \frac{10}{4} = 0$

$x_{1/2} = -\frac{9}{4} \pm \sqrt{\frac{81}{16} + \frac{40}{16}} = x_{1/2} = -\frac{9}{4} \pm \sqrt{\frac{121}{16}}$

$x_1 = -\frac{9}{4} - \frac{11}{4} = -\frac{20}{4} = -5$ $\quad \mathbb{L} = \{-5; \frac{1}{2}\}$

$x_2 = -\frac{9}{4} + \frac{11}{4} = \frac{2}{4} = \frac{1}{2}$

f) $\frac{5x^2 - 40x + 11}{(x-3)(2x+2)} + \frac{x-1}{2x+2} = \frac{x-7}{x-3}$ $\quad \mathbb{D} = \mathbb{R} \setminus \{3; -1\}$ $\quad | \cdot (x-3)(2x+2)$

$5x^2 - 40x + 11 + (x-1) \cdot (x-3) = (x-7)(2x+2)$

$5x^2 - 40x + 11 + x^2 - 3x - x + 3 = 2x^2 + 2x - 14x - 14$

$6x^2 - 44x + 14 = 2x^2 - 12x - 14$ $\quad | - 2x^2 + 12x + 14$

$4x^2 - 32x + 28 = 0$ $\quad | : 4$

$x^2 - 8x + 7 = 0$

$x_{1/2} = 4 \pm \sqrt{16 - 7}$

$x_1 = 4 - 3 = 1$ $\quad \mathbb{L} = \{1; 7\}$

$x_2 = 4 + 3 = 7$

Lösungen Bruchgleichungen (11)

g) $\frac{4x^2-34x-228}{(x-10)(x-2)} = \frac{x+3}{x-2} - \frac{-2x+1}{x-10}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{10;2\}$ $\quad |\cdot(x-10)(x-2)$

$4x^2-34x-228 = (x+3)\cdot(x-10) \ominus (2x+1)\cdot(x-2)$

$4x^2-34x-228 = x^2-10x+3x-30 \ominus [2x^2-4x+1x-2]$

$4x^2-34x-228 = x^2-10x+3x-30 \underline{-2x^2+4x-1x+2}$

$4x^2-34x-228 = -x^2-4x-28$ $\quad |+x^2+4x+28$

$5x^2-30x-200 = 0$ $\quad |:5$

$x^2-6x-40 = 0$

$x_{1/2} = 3 \pm \sqrt{9+40} \Rightarrow x_1 = 3-7 = -4$

$x_2 = 3+7 = 10$

$\mathbb{L} = \{-4\}$ da $10 \notin \mathbb{D}$

Das Minus vor der Klammer bezieht sich auf alles hintendran, deshalb [Klammer setzen]!

⇒ alle Vorzeichen in der Klammer umdrehen!

h) $\frac{4x+6}{x+6} - \frac{x-18}{x^2-36} = \frac{x-5}{x-6}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{-6;6\}$ $\quad |(x+6)(x-6)$ $\quad$ NR: $(x+6)(x-6) = x^2-36$

$(4x+6)\cdot(x-6) - (x-18) = (x-5)\cdot(x+6)$

$4x^2-24x+6x-36-x+18 = x^2+6x-5x-30$

$4x^2-19x-18 = x^2+x-30$ $\quad |-x^2-x+30$

$3x^2-20x+12 = 0$ $\quad |:3$

$x^2-\frac{20}{3}x+4 = 0$

$x_{1/2} = \frac{10}{3} \pm \sqrt{\frac{100}{9}-4} = \frac{10}{3} \pm \sqrt{\frac{64}{9}}$ $\quad$ NR: $4 = \frac{36}{9}$

$x_1 = \frac{10}{3} - \frac{8}{3} = \frac{2}{3}$

$x_2 = \frac{10}{3} + \frac{8}{3} = \frac{18}{3} = 6$

$\mathbb{L} = \{\frac{2}{3}\}$ da $6 \notin \mathbb{D}$

i) $\frac{9x^2+20x-69}{(2x-4)(x+5)} - \frac{x+1}{x-5} = \frac{x-1}{2x-4}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{5;2\}$ $\quad |\cdot(2x-4)(x+5)$

$9x^2+20x-69 - (x+1)\cdot(2x-4) = (x-1)\cdot(x+5)$

$9x^2+20x-69 - [2x^2-4x+2x-4] = x^2+5x-x-5$

$9x^2+20x-69-2x^2+4x-2x+4 = x^2+4x-5$

$7x^2+22x-65 = x^2+4x-5 \quad |-x^2-4x+5$

$6x^2+18x-60 = 0 \quad |:6$

$x^2+3x-10 = 0$

$x_{1/2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4}+\frac{40}{4}} = -\frac{3}{2} \pm \sqrt{\frac{49}{4}}$

$x_1 = -\frac{3}{2} - \frac{7}{2} = -\frac{10}{2} = -5$

$x_2 = -\frac{3}{2} + \frac{7}{2} = \frac{4}{2} = 2$

$\mathbb{L} = \{-5\}$ da $2 \notin \mathbb{D}$

j) $\frac{4x-8}{x^2-4x+4} = \frac{x+4}{2x-4} - \frac{2x-1}{x-2}$ $\quad \mathbb{D} = \mathbb{R}\setminus\{2\}$ $\quad |\cdot 2\cdot(x-2)^2$

NR: $x^2-4x+4 = (x-2)^2$

$2x-4 = 2\cdot(x-2)$

$\frac{(4x-8)\cdot 2\cdot(x-2)^2}{x^2-4x+4} = \frac{(x+4)\cdot 2\cdot(x-2)(x-2)}{2x-4} - \frac{(2x-1)\cdot 2\cdot(x-2)(x-2)}{x-2}$

$8x-16 = x^2-2x+4x-8 - [4x^2-8x-2x+4]$

$8x-16 = x^2+2x-8 \quad -4x^2+8x+2x-4$

$8x-16 = -3x^2+12x-12 \quad |+3x^2-12x+12$

$3x^2-4x-4 = 0 \quad |:3$

$x^2-\frac{4}{3}x-\frac{4}{3} = 0$

$x_{1/2} = +\frac{2}{3} \pm \sqrt{\frac{4}{9}+\frac{4}{3}} = +\frac{2}{3} \pm \sqrt{\frac{4}{9}+\frac{12}{9}}$

$x_1 = +\frac{2}{3} - \frac{4}{3} = -\frac{2}{3}$

$x_2 = +\frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

$\mathbb{L} = \{-\frac{2}{3}\}$ da $2 \notin \mathbb{D}$