

# Beispielrechnung Bruchgleichungen (10)

$$\mathbb{D} = \mathbb{R} \setminus \{-2\}$$

$$a) \frac{x^2 - 8}{x + 2} - 4 = \frac{2x}{x + 2} \quad | \cdot (x + 2)$$

$$\frac{x^2 - 8 \cdot (x + 2)}{x + 2} - 4 \cdot (x + 2) = \frac{2x \cdot (x + 2)}{x + 2} \quad [\text{Zürzen}]$$

$$x^2 - 8 - 4x - 8 = 2x \quad [\text{Zusammenfassen}]$$

$$x^2 - 4x - 16 = 2x \quad | -2x$$

$$x^2 - 6x - 16 = 0$$

$$x_{1/2} = 3 \pm \sqrt{9 + 16}$$

$$x_1 = 3 - 5 = -2$$

$$x_2 = 3 + 5 = 8$$

$$\underline{\underline{\mathbb{L} = \{8\}}}$$

Adh. Definiertmenge

$$b) 2x \cdot \frac{x-1}{x+3} + 1 = \frac{4}{x+3} \quad \mathbb{D} = \mathbb{R} \setminus \{-3\} \quad | \cdot (x+3)$$

$$\frac{2x \cdot (x-1) \cdot (x+3)}{x+3} + 1 \cdot (x+3) = \frac{4 \cdot (x+3)}{x+3}$$

$$2x^2 - 2x + x + 3 = 4$$

$$2x^2 - x + 3 = 4 \quad | -4$$

$$2x^2 - x - 1 = 0 \quad | :2$$

$$x^2 - \frac{1}{2}x - \frac{1}{2} = 0$$

$$x_{1/2} = + \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{1}{2}}$$

$$x_{1/2} = + \frac{1}{4} \pm \sqrt{\frac{9}{16}}$$

$$x_1 = + \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$x_2 = + \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

$$\underline{\underline{\mathbb{L} = \{-\frac{1}{2}; 1\}}}$$

$$c) 1 + \frac{2}{x} = \frac{3}{x-2} \quad \mathbb{D} = \mathbb{R} \setminus \{0; 2\} \quad | \cdot x \cdot (x-2)$$

$$x \cdot (x-2) + 2 \cdot (x-2) = 3 \cdot x$$

$$x^2 - 2x + 2x - 4 = 3x$$

$$x^2 - 4 = 3x \quad | -3x$$

$$x^2 - 3x - 4 = 0$$

$$x_{1/2} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + 4} \quad 4 = \frac{16}{4}$$

$$x_1 = \frac{3}{2} - \frac{5}{2} = -\frac{2}{2} = -1$$

$$x_2 = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$$

$$\underline{\underline{\mathbb{L} = \{-1; 4\}}}$$

$$1 \cdot x \cdot (x-2) + \frac{2 \cdot x \cdot (x-2)}{x} = \frac{3 \cdot x \cdot (x-2)}{x-2}$$



Bruchgleichungen (10)

d)  $\frac{5}{x} - \frac{10}{x-3} = 10$   $\mathbb{D} = \mathbb{R} \setminus \{0; 3\}$   
 $| \cdot x \cdot (x-3)$

$5 \cdot (x-3) - 10 \cdot x = 10 \cdot x \cdot (x-3)$   
 $5x - 15 - 10x = 10x^2 - 30x$   
 $-5x - 15 = 10x^2 - 30x \quad | +5x + 15$   
 $0 = 10x^2 - 25x + 15 \quad | : 5$   
 $0 = 2x^2 - 5x + 3$

$x_{1/2} = \frac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot 3}}{2 \cdot 2}$

$x_{1/2} = \frac{5 \pm 1}{4}$

$x_1 = \frac{5-1}{4} = \frac{4}{4} = 1$

$x_2 = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2}$

$\mathbb{L} = \left\{ 1; \frac{3}{2} \right\}$

$\frac{5 \cdot \cancel{x} \cdot (x-3)}{\cancel{x}} - \frac{10 \cdot \cancel{x} \cdot (x-3)}{\cancel{x-3}} = 10 \cdot x \cdot (x-3)$

f)  $\frac{3x-4}{x} = -\frac{6}{x+1} - 1$   $\mathbb{D} = \mathbb{R} \setminus \{0; -1\}$   
 $| \cdot x \cdot (x+1)$

$(3x-4) \cdot (x+1) = -6 \cdot x - x \cdot (x+1)$   
 $3x^2 + 3x - 4x - 4 = -6x - x^2 - 1x$   
 $3x^2 - x - 4 = -x^2 - 7x \quad | +x^2 + 7x$   
 $4x^2 + 6x - 4 = 0 \quad | : 4$   
 $x^2 + \frac{3}{2}x - 1 = 0$

$x_{1/2} = -\frac{3}{4} \pm \sqrt{\frac{9}{16} + \frac{16}{16}}$

$x_1 = -\frac{3}{4} - \frac{5}{4} = -\frac{8}{4} = -2$

$x_2 = -\frac{3}{4} + \frac{5}{4} = \frac{2}{4} = \frac{1}{2}$

$\mathbb{L} = \left\{ -2; \frac{1}{2} \right\}$

$\frac{(3x-4) \cdot \cancel{x} \cdot (x+1)}{\cancel{x}} = \frac{-6 \cdot \cancel{x} \cdot (x+1)}{\cancel{x+1}} - 1 \cdot x \cdot (x+1)$

h)  $\frac{x^2+4}{x} - \frac{x+2}{2x} = \frac{3x^2+12}{4x}$   $\mathbb{D} = \mathbb{R} \setminus \{0\}$   
 $| \cdot 4x$

$(x^2+4) \cdot 4 - (x+2) \cdot 2 = 3x^2+12$   
 $4x^2+16 - 2x-4 = 3x^2+12$   
 $4x^2-2x+12 = 3x^2+12 \quad | -3x^2-12$   
 $x^2-2x = 0$   
 $x \cdot (x-2) = 0$

$\rightarrow x_1 = 0 \quad x_2 = 2 \quad \mathbb{L} = \{2\}$   
 da  $x \notin \mathbb{D}$

e)  $\frac{x}{x+3} - 3 = \frac{4-3x}{x}$   $\mathbb{D} = \mathbb{R} \setminus \{0; -3\}$   
 $| \cdot x \cdot (x+3)$

$x \cdot x - 3 \cdot x \cdot (x+3) = (4-3x) \cdot (x+3)$   
 $x^2 - 3x^2 - 9x = 4x + 12 - 3x^2 - 9x \quad | -x^2 + 5x$   
 $x^2 - 4x - 12 = 0$

$x_{1/2} = 2 \pm \sqrt{4+12}$

$x_1 = 2 - 4 = -2$

$x_2 = 2 + 4 = 6 \quad \mathbb{L} = \{-2; 6\}$

$\frac{x \cdot \cancel{x} \cdot (x+3)}{\cancel{x+3}} - 3 \cdot x \cdot (x+3) = \frac{(4-3x) \cdot \cancel{x} \cdot (x+3)}{\cancel{x}}$

\* Klammer selbst setzen, weil es zusammengehört!

g)  $\frac{x^2-x+2}{12x} - \frac{x+2}{3x} - \frac{x+1}{4x} = 0$   $\mathbb{D} = \mathbb{R} \setminus \{0\}$   
 $| \cdot 3 \cdot 4x$

$x^2 - x + 2 - (x+2) \cdot 4 - (x+1) \cdot 3 = 0$   
 $x^2 - x + 2 - 4x - 8 - 3x - 3 = 0$   
 $x^2 - 8x - 9 = 0$

$x_{1/2} = 4 \pm \sqrt{16+9}$

$x_1 = 4 - 5 = -1$

$x_2 = 4 + 5 = 9 \quad \mathbb{L} = \{-1; 9\}$

$\frac{x^2-x+2 \cdot \cancel{3 \cdot 4x}}{\cancel{12x}} - \frac{(x+2) \cdot \cancel{3 \cdot 4x}}{\cancel{3x}} - \frac{(x+1) \cdot \cancel{3 \cdot 4x}}{\cancel{4x}} = 0$